The Auditor’s Reputation and its Effect on Audit Quality and Audit Premia

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Abstract
In this paper, we emphasize the importance of reputational concerns for parameters of the audit process. Based on Bayesian belief updating, we model a concave reputation function, which represents auditor’s history of successful/unsuccessful signals and indicates the potential reputational gains/losses for an auditor at a given point in time. We show that audit quality does not always increase in audit reputation, as commonly suggested, but has a U-shaped form. As a consequence, the client does not automatically prefer high reputation in an auditor, but sometimes picks the auditor with the smaller reputation. Based on this finding, we decompose the components of a possible Big 4 premium and identify that it is only quality driven when the negotiation power lies mostly with the auditor and not with the client firm.

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1 Introduction

Auditing is a profession mostly based on trust. Capital markets rely on the correct verification of financial statements in order to improve their decision making process. Similar to a doctor’s appointment, it is difficult to conclude whether the task has been performed appropriately because the work effort is not easily observable. This characteristic pushes the auditor’s reputation in a spotlight when it comes to acquiring new engagements. However, the research on audit reputation has been scarce. Except Datar and Alles (1999) and Corona and Randhawa (2010), analytical research either abstains from modeling reputation separately or mostly considers reputation losses which are linear and additive to a potential litigation payment (Ewert 1999, and Bigus 2006). Our model suggests that the reputation function is more likely concave based on the idea of Bayesian belief updating. We exploit this concavity and examine the effects of reputation gains and losses on audit quality as well as their subsequent effect on audit rents and audit premia. We show that both audit quality and the firm value of a client do not always increase in audit reputation, but have a U-shaped form. As a consequence, the client does not necessarily prefer higher reputation in an auditor.

We model a binary single-period investment game where a company hires on of two possible auditors, who differ with respect to their ex-ante reputation and their accumulated wealth. The signal from the verification process is then used to support an outside investor’s decision to invest in the company’s project. The auditor is subject to reputational concerns based on Bayesian belief updating, which we understand in the following way. In the beginning of a given period, an auditor’s reputation is the sum of her successful signals minus her unsuccessful signals of the past, where a
successful (unsuccessful) signal is an audit in which a low economic state of the company is correctly (incorrectly) represented in the audit report. The more successful signals an auditor accumulates, the lower the marginal gain on every future signal. Similarly, a reputation loss for a given mandate is always bigger than a reputation gain. Both the reputation gain and loss depend on the size or complexity of the client in the sense that a successful (unsuccessful) audit of a large client has a bigger positive (negative) effect on the auditor. For example, while a minor accounting scandal on a medium-sized client dampens a Big 4 auditor’s reputation marginally for a few weeks, it can cause damages beyond repair for a smaller local auditor. A big auditor is only forced out of business in extreme scandals such as Arthur Anderson in the Enron case.\footnote{At the time of the scandal, Enron was the 7th largest company in the US.} Similarly from a reputational gains perspective, Asthana and Kalkar (2014) find evidence that the auditor’s reputational benefits are bigger for the first S&P 500 client than for any further S&P 500 client.

The concavity of the reputation function also implies that a small auditor suffers a stronger loss from reputation than a large auditor. Although there is still a lack of evidence for this concept (or the evidence opposing this concept), there are examples in practice such as the Parmalat Scandal of 2003. Grant Thornton was replaced by Deloitte as an auditor on conglomerate level in 1999 as part of a mandatory rotation. By the year 2003, both audit firms audited approximately 50% of the total company (including separate company financial statements). As a consequence of the scandal, Grant Thornton International and Deloitte Touche Tohmatsu settled a lawsuit in the US for a total of $15 million (GTI $6.5 million and Deloitte $8.5 million). This settlement indicates that both auditors are to blame in equal parts for the failed audit in terms of litigation. However, the smaller firm Grant Thornton banned its Italian

\footnote{At the time of the scandal, Enron was the 7th largest company in the US.}
branch from the audit network (now Italaudit SpA) immediately after the scandal to control damage while Deloitte continued business as usual. Hence, the smaller auditor, Grant Thornton, did no longer believe it could acquire clients in Italy and seemed to have suffered the bigger reputational damage.\(^2\)

This concept of bigger reputation losses for smaller auditors contrasts the widely spread reasoning from DeAngelo (1981). She claims that bigger auditors have ‘more to lose’ since the loss of reputation following an audit failure in one audit engagement affects the faith of other clients in the auditor’s work. In DeAngelo (1981), auditor size is associated with a higher number of client engagements and ergo a higher amount of audit rents, which incentivizes bigger auditors to work harder. However, DeAngelo (1981) does not consider the auditor’s risk of losing her entire reputation and being forced to exit the market, which is much stronger for smaller auditors given that client size is fixed. While a negative effect of reputation on other clients certainly harms a bigger audit firm momentarily, it has the chance to recover and to continue generating positive rents in contrast to a small audit firm forced to exit the market which can be seen in the Parmalat case. This is why we argue that the relative costs of reputation loss and thereby the incentives to work harder are stronger for auditors with a smaller ex ante reputation.

\(^2\)Other prominent cases, in which a small auditor suffered a reputation loss beyond repair, are M&A auditees in the audit of Let’s GOWEX SA, Spain, in 2014 or GWP in the audit of Co op, Germany, in 1988. In the case of the bigger Japanese auditor ChuoAoyama, affiliated with PwC network, PwC tried a turnaround after irregularity at the audit of Kanebo, 2003-2006. However, after the second incident at Nikko Cordial within a short period, 2006, ChuoAoyama eventually had to shut down (Skinner and Srinivasan 2012). These cases are all consistent with our modeling approach. While the smaller auditors immediately had to leave the market, the Big audit network still tried the turnaround after the first scandal.
As a result of the U-shaped impact of audit reputation on firm value, it can be better for a client if an auditor has a smaller reputation, as long as she is still capable to provide the required resources to complete the audit for the specific client adequately. The reason for this result is that a smaller reputation auditor faces the danger of losing almost her entire reputation and possibly has to go out of business. This motivates her to exert a higher audit effort and to provide a higher audit quality, which can exceed the audit quality of an auditor with a higher reputation given the litigation concerns are not too strong in relation to the reputation concerns.

In addition to the difference in reputation, we also consider a wealth difference between the auditors, whereby the auditor with the higher reputation has also the higher wealth. The reason for this assumption is that higher reputation implies that the auditor has accumulated more successful signals in the past - and thereby potentially more rents - which increase the total wealth. We call this auditor with the higher reputation and wealth also the Big 4 auditor. The wealth difference matters if one (or both) auditor(s) are not able to cover damage payments from investor lawsuits whenever a failure in the audit process occurs. The wealth restriction changes the smaller auditor’s trade off with respect to three factors. 1) The exposure to expected litigation decreases, which dampens the incentive to choose high audit quality. 2) The insurance function of auditing for the investor decreases, which raises the cost of capital. 3) The expected audit cost decline due to lower levels of audit quality and lower expected litigation. Factor 1) and 2) always dominate factor (3), which means that the wealth difference always works in favor of the Big 4 auditor, contrasting the previously discussed reputation effect which can work in both directions depending on the respective parameters of the engagement. In sum, our model shows that it is not obvious which auditor provides the higher audit quality and which one would be the
optimal choice for the client firm.

This result challenges empirical studies based on the former mentioned reasoning from DeAngelo (1981), which simply assume that the Big 4 auditor always provides higher audit quality (Becker et al. 1998; Choi and Wong 2007; Behn et al. 2008; Francis and Wang 2008). We specify our results in the sense that a non-Big 4 auditor may provide an equal or even higher audit quality than a Big 4 auditor, especially when litigation concerns are lower. This result is also in line with empirical studies. Khurana and Raman (2004) and Maijoor and Vanstraelen (2006) find that Big 4 auditors are associated with a higher audit quality in the US, which is generally known for its strong litigation regime, but not in Australia, Canada or European countries, which are all considered as lower litigation countries. The generally mixed findings with respect to Big 4 audit quality address the fact that the mere size of an auditor might not be an adequate proxy for audit quality. Lawrence et al. (2011) analyze this concern in a recent paper and show that the empirical results regarding audit quality could describe significant difference between the audited clients, e.g. in terms of client size, rather than differences between Big 4 vs non-Big 4 auditors. The source for the overall mixed empirical evidence could be connected to the fact that the audit quality does not necessarily increase in audit reputation and also the smaller auditor is sometimes able to provide better audit quality than the bigger competitor. This is only possible if the differences between the auditors are moderate. Thus, a medium sized auditor might be able to compete with a Big 4 auditor for an audit engagement given that the client is still manageable in size. A small local office however can never challenge a Big 4 auditor according to our model.

The Big 4 premium is a topic closely related to Big 4 audit quality. Empirical research has been after the question whether there is a fee premium, which a client
company is willing to pay in order to hire a Big 4 auditor and under which conditions such a premium can be established. Thereby, the literature has generated a lot of mixed evidence (e.g. Ireland and Lennox 2002, Hay et al. 2006, Choi et al. 2008, Francis and Wang 2008, Johnstone et al. 2014). Our model contributes to this discussion in that we determine the underlying components of the Big 4 premium. The two components are 1) the difference in net audit costs between Big 4 and non-Big 4 auditors and 2) the difference in the client company’s cost of capital with each auditor. Both are weighted according to the relative bargaining power between the engaged auditor and the client company. If the auditor (client) has all the bargaining power, the premium solely contains the cost of capital (audit net cost) component. Therefore, our model shows that the fee premium is only a good proxy for audit quality if the auditor has indeed all or most of the bargaining power.

In our model, the client size or complexity also plays a major role since the reputational gains or losses increase in client size. A smaller auditor could potentially be a better fit for a large client as long as she can still provide the resources and the litigation concerns are not too strong. However, the bigger the client, the smaller the selection of auditors which are capable of providing the resources for an adequate audit. Consequently, some large conglomerates can, indeed, only pick a Big 4 auditor. In these cases, audit rents are the highest because the off-equilibrium option to pick the smaller auditor is missing and the Big 4 auditor’s benefit over the next best option (no auditor) increases. These rents are potentially the source of the dominance of Big 4 auditors in the market and their competitive advantage over smaller auditors, not their higher reputation itself.

As mentioned above, there are two papers in the analytical audit literature which focus on reputation in more detail and develop reputation endogenously. Datar and
Alles (1999) show that reputational concerns have a disciplining function and allow the auditor to cut back on testing without increasing the risk of manipulation. In Corona and Randhawa (2010), reputation can have the exact opposite effect. In a single interaction, reputation provides the right incentives, but if the auditor is concerned about a long-term engagement it encourages misreporting from the auditor. The reason is that fraud detected in later periods highlights audit failure in earlier periods and impairs the auditor’s reputation. In an environment with high penalties and a small frequency of manipulation, the auditor might conceal fraud in order to hide her error and avoid harming her reputation. Different to these papers, we assume that the auditor always reports truthfully. As a result, reputation is solely based on ability and not on reporting behavior. In addition, we do not derive reputation endogenously but we provide solid reasons why an auditor’s reputation should be a concave function of positive and negative signals. Thereby, we condense the auditor’s history in a single function and develop our result in a single shot game.

Reputation is also important in other markets. Early economic literature, such as Shapiro (1983), shows that it is important to build up reputation in order to sell high quality goods at a high price. This requires an investment in reputation below costs. We also have this investment in reputation since the auditor is willing to abstain from monetary fees, which she would receive absent the reputation concerns, in order to gain reputation. What is different to Shapiro (1983) is that our model has an additional moral hazard problem. In his paper, the buyer knows that the goods will have a high quality if the seller has a high reputation. In our model, a high reputation auditor only has the possibility to provide high quality services due to his superior efficiency, but the actual work input is uncertain and depends on a bundle of incentives, which also allows low reputation auditors to provide superior
audit quality.

Reputation in consumer markets is also considered empirically. The feedback function of the internet platform eBay provides excellent data and hence, is used by many researchers to conduct studies. Most prominently, Cabral and Hortacsu (2010) find that buyers react to reputation. Sellers are more likely to exit the market when they have low reputation and they receive negative feedback more frequently before they exit. Although the findings stem from a market fairly different from audit reputation, they are consistent with our idea of a concave reputation function.

The remainder of the paper is organized as follows. Section 2 outlines the model. The equilibrium strategies are derived and discussed in the following sections and comparative statistics of the equilibrium strategies are conducted. Section 3 analyses the equilibrium audit quality, Section 4 the equilibrium cost of capital and Section 5 the equilibrium audit fee. Section 6 concludes. All proofs are in Appendix A and B.

2 Model

We consider a binary single-period investment game with three risk-neutral parties: a company, an auditor, and investors. The company wants to carry out a project which is good $g$ (bad $b$), with probability $p$ ($1 - p$), but the type of the project is not observable. The project requires an investment of $I$ and yields a cash flow of $\Pi_s$ ($\Pi_f$) in case the project is successful (fails). For simplicity, we normalize cash flow $\Pi_f$ to zero and refer to the high cash flow as $\Pi_s = \Pi$ with $\Pi > I$.

Timeline. The model has a simple structure with three dates: 0, 1, 2, and 3. At date 0, nature draws a project type, $t \in \{g, b\}$. At date 1, the company hires an auditor and pays him fee, $F$. At date 2, the auditor chooses audit effort, $e$, and
releases a report, \( \hat{t} \in \{ \hat{g}, \hat{b} \} \), which is informative about the project type. Based on this report, the investors decide whether to finance the project. At date 3, the payoffs are realized and investors can sue the auditor for issuing a misleading report.

**Audit Market.** The audit market consists of two auditors \( i \in \{ l, h \} \), who differ in ex ante reputation with \( r_l < r_h \) and their ex ante wealth \( W_l < W_h \). The auditor with the higher reputation also has the higher wealth, since she has accumulated more successful signals (and thereby potentially more rents) in the past. We call this auditor high reputation auditor or Big 4 auditor. If hired, each auditor chooses audit effort \( e_i \in [0, 1] \), which also determines the audit quality. Variable \( e_i \) reflects the probability that the auditor reports \( \hat{b} \) given that the project type is really \( b \). In line with audit literature, we also exclude type I errors of the auditor, which means that the auditor never misclassifies a truly good project. We assume further that the auditor reports always truthfully.

**Audit Costs.** The audit costs are a convex function with respect to audit effort, \( C(e_i, r_i) \), with \( \frac{dC(e_i, r_i)}{de_i} > 0 \) and \( \frac{d^2C(e_i, r_i)}{de_i^2} > 0 \). Reputation directly affects the audit costs due to several reasons. High reputation audit firms have conducted more successful audits in the past and have accumulated more experience which are connected with lower costs. In addition, high reputation auditors are more attractive to young talents and have more resources to spend on recruiting, which also contributes to more efficient audits (Dopuch and Simunic 1980). Further, high reputation audit firms are in practice often associated with a larger size (e.g. Big 4 audit firms) and therefore they are better able to capitalize on economies of scale. Consequently, audit costs decrease with higher reputation, \( \frac{dC(e_i, r_i)}{dr_i} < 0 \) and \( \frac{d^2C(e_i, r_i)}{dr_i^2} > 0 \). We further consider the audit cost parameter, \( k > 0 \), which reflects a basic audit technology underlying all auditors independent of the reputation. A higher efficiency is associated with smaller
values of \( k \). We assume that the audit costs parameter is sufficiently high, \( k \geq k^{\text{min}} \), to ensure that the effort level, \( e_i \), does not exceed one.\(^3\) To be more specific, the presumed audit costs function in the model analysis has the form \( C(e_i, r_i) = \frac{k}{2r_i}e_i^2 \).

**Audit Incentives.** The auditor is incentivized to exert effort through legal liability and reputation concerns. The incentives provided by legal liability refer to the costs that occur in connection with a lawsuit in case of an incorrect audit opinion. Thereby, we assume a strict legal liability system where the auditor is always liable for not detecting a bad project. This liability payment, \( D \), is a transfer payment to the investors with \( D < I \). The level of \( D \) is also important with respect to a possible wealth effect. If the wealth of both auditors is higher than the potential litigation payment, \( W_h > W_l > D \), the wealth difference does not affect the optimal solution of our model. If, on the other hand, the wealth of one (or both) auditors is not sufficient to cover the litigation payment, \( W_l < D < W_h \), the wealth difference affects our results.\(^4\)

We understand the reputation of an auditor based on Bayesian belief updating as the history of good and bad signals in the beginning of a given period. Appendix B provides an example how audit reputation relates to the properties of Bayesian belief updating.\(^5\) The result of this derivation is a concave function which depends on the ex ante reputation of the auditor, \( r_i > 0 \), and the reputation attached to the current engagement, \( \tau \). This reputation is larger the bigger the size of the client, \( \frac{d\tau(s)}{ds} > 0 \),

\[^3\] \( k^{\text{min}} = (1 - p)r_i(D + R_i^+ + R_i^-) \) results from the optimal audit effort, \( e_i^* \), from equation (5).

\[^4\] Contrary to the loss of reputation, the loss of wealth due to possible litigation is linear and influences both auditors in the same way. To our knowledge, there is no case where the auditor was forced out of business due to insufficient funds.

\[^5\] We do not claim that Appendix B can be applied to our main model one-to-one. We only want to give an example of how Bayesian updating can lead to the same class of concave function.
for example⁶

reputation gain: \( R_i^+ = \sqrt{r_i + \overline{r}(s)} - \sqrt{r_i} \)

reputation loss: \( R_i^- = \sqrt{r_i} - \sqrt{r_i - \overline{r}(s)} \)

\( R_i^+ \) and \( R_i^- \) represent the total effect of reputation on the utility of the respective auditor in case of a successful or unsuccessful audit. This includes the probability update of the market participants in terms of future likelihood of successful signals as well as a gain or loss in reputational capital. The underlying concave reputation function is associated with the following properties: First, a company with high reputation can gain and lose less reputation than a company with low reputation, \( \frac{dR_i^+}{dr_i} < 0 \) and \( \frac{dR_i^-}{dr_i} < 0 \). The idea is that investors are more uncertain about the quality of a low reputation auditor, and therefore, an additional positive or negative signal has a higher impact on the investor’s belief. In addition, we argue that the relative loss of reputation for a low reputation auditor is stronger since she might be forced to leave the market, while a high reputation auditor might lose rents from other clients momentarily but has the ability to recover. This assumption is in contrast to DeAngelo (1981) who argues that the bigger auditor has more to lose.

Second, the reputation gain and loss increases in \( \overline{r}(s) \), \( \frac{dR_i^+}{dr(s)} > 0 \) and \( \frac{dR_i^-}{dr(s)} > 0 \). Therefore, if the engagement is more important due to the size of the client, the auditor can gain and lose more reputation. We relate this to the higher public awareness that is connected with larger clients.

Third, the potential reputation loss is larger than the potential reputation gain.

⁶Note, that \( R^+ \) and \( R^- \) represent the changes (gain or loss in reputation) of an concave function and therefore do not have the concave properties themselves. The underlying concave function in this example is \( R = \sqrt{r} \).
due to the concavity of the reputation function. This means that the auditor can lose more reputation with an audit failure than she can potentially gain from auditing the client.

Finally, the auditor cannot lose more reputation with the current engagement than it has prior to the engagement, $r_i - \bar{r}(s) > 0$. This also means that an auditor needs to gain repuation, experience and collect resources to be able to audit larger clients. A small local audit office with a low ex ante reputation is not able to audit a large conglomerate and hence $r_i > \bar{r}(s)$ has to be meet such that the auditor is considered for engagement by the company. Further note that the auditor does not participate in windfall gains. This means she does not gain reputation in an "easy" audit where the project type is good anyways.

**Investors.** The investors will only invest if they receive a good signal in the audit report, $\hat{g}$. Only if the report is high, the project can be successful and the investor has a chance of receiving a predetermined share $\alpha_i \in [0, 1]$ of the profits. In case of an undetected bad type project, the investors can sue the auditor and receive a damage payment $D$.

**Utilities of the players.** In summary, the utilities of the company, $U^C$, of the auditor, $U^A$, and of the investors, $U^I$, are given by

\begin{align*}
U^C_i &= p(1 - \alpha_i)\Pi_i - F_i, \\
U^A_i &= F_i - \frac{k}{2r_i}e_i^2 + (1 - p)e_iR_i^+ - (1 - p)(1 - \alpha_i)(\min\{W_i, D\} + R_i^-), \quad \text{and} \\
U^I_i &= p(\alpha_i\Pi_i - I) + (1 - p)(1 - e_i)(\min\{W_i, D\} - I).
\end{align*}

\[^7\text{This assumption is also mathematically relevant because otherwise the term } \sqrt{r_i - \bar{r}(s)} \text{ would not be defined.}\]

\[^8\text{This assumption does not change the qualitative results of this paper.}\]
3 Optimal Audit Quality

We begin the analysis by determining the auditor’s optimal audit quality level. The auditor receives a fixed non-contingent fee for her services. She is interested to minimize her net costs, consisting of the direct effort costs, the expected litigation costs and the reputation losses and gains. The auditor participates in the contract with the client company if this net costs are at least covered. Any fee exceeding the net costs will not incentivize the auditor to exert more effort since the effort is not easily observable ex post which means that the auditor would just shirk and consume the additional rents. Therefore, the auditor’s incentive problem can be expressed as

$$\min_{e_i} \frac{k}{2r_i} e_i^2 + (1-p)(1-e_i)(\min\{W_i, D\} + R_i^-) - (1-p)e_iR_i^+$$  \hspace{1cm} (4)$$

If hired, each auditor $i \in \{l, h\}$ minimizes her costs based on the former mentioned factors: direct effort costs, $\frac{k}{2r_i} e_i^2$, the threat of litigation, $(1-p)(1-e_i)D$, the potential reputation losses, $(1-p)(1-e_i)R_i^-$, and reputation gains, $-(1-p)e_iR_i^+$. The incentives from reputation gains serve as a substitute to the monetary incentives from the fixed audit fee. The equilibrium audit quality $e_i^*$ is given by

$$e_i^* = \frac{(1-p)r_i(\min\{W_i, D\} + R_i^+ + R_i^-)}{k}$$  \hspace{1cm} (5)$$

Suppose for a moment that the wealth difference between the auditors does not play a role and both are able to cover the damage payment from litigation, $D < W_l < W_h$. Therefore, the auditors only differ with respect to their ex ante reputation. In what follows, we focus on the influence of reputation on the optimal audit quality, which is characterized by the following equation:

$$\frac{de_i^*}{dr_i} = \frac{(1-p)}{k} \left[ (D + R_i^+ + R_i^-) + r_i \left( \frac{dR_i^+}{dr_i} + \frac{dR_i^-}{dr_i} \right) \right].$$  \hspace{1cm} (6)$$

\begin{align*}
\text{direct cost effect} & > 0 \\
\text{incentive effect} & < 0
\end{align*}
Equation (6) directly leads to the proposition.

**Proposition 1** There is a threshold $\hat{r}_e$ such that an increase in ex ante reputation, $r_i$,

(i) decreases the equilibrium audit effort, $e_i^*$, for all $r_i < \hat{r}_e$, and

(ii) increases the equilibrium audit effort, $e_i^*$, for all $r_i > \hat{r}_e$.

Ex ante reputation affects the optimal audit effort in two opposing ways: On the one hand, a higher reputation is connected with higher audit efficiency due to learning, recruiting and economies of scale effects. This direct cost effect of reputation enables the auditor to spend higher audit effort with increasing cost efficiency. On the other hand, according to our concave reputation function, a higher reputation is associated with a lower potential of gaining or losing reputation. This incentive effect induces the auditor to choose lower audit quality with increasing reputation and is negative, $\frac{dR^-}{dr_i} < 0$ and $\frac{dR^+}{dr_i} < 0$. Overall, the total effect is ambiguous. For auditor’s with relatively low ex-ante reputation, the incentive effect outweighs the direct cost effect. However, as the reputation increases, the incentives from reputation gains and losses decrease while the direct cost effect increases, until the latter one dominates for all $r_i > \hat{r}_e$. The threshold, $\hat{r}_e$, marks the minimum of the function $e_i^*(r_i)$. The resulting U-shaped form of Proposition 1 leads to the observation that auditors with higher ex-ante reputation do not necessarily offer a higher audit quality.

Note that there can be reputation functions that meet our criteria from Bayesian belief updating but where the cost effect always dominates. Therefore, the results of our model apply only to economies where the reputation incentives are strong enough to matter. This is also closely related to the role of the litigation environment that is described in the next paragraph.
Equation (6) provides another interesting insight. Higher litigation damage payment, $D$, benefits Big 4 auditors. The reason for this is that a higher litigation only drives the direct cost effect and thereby crowds out the reputation incentive effect, which is generally larger in size for auditors with a lower ex-ante reputation (see equation (6)).

In addition, we now introduce the case when the wealth difference matters, i.e. $W_l < D < W_h$. The resulting audit quality levels are

\[ e^*_{h} = \frac{(1-p)r_h(D + R^+_h + R^-_h)}{k} \text{ and } e^*_{l} = \frac{(1-p)r_l(W_l + R^+_l + R^-_l)}{k} \]

with \[
\Delta e = e^*_h - e^*_l = \frac{(1-p)}{k} \left[ r_h(D + R^+_h + R^-_h) - r_l(W_l + R^+_l + R^-_l) \right]
\] (7)

If $\Delta e > 0$, the Big 4 auditor provides a higher audit quality. The results are summarized in the following lemma:

**Lemma 1** If the wealth effect matters, $W_l < D < W_h$, the difference in audit quality, $\Delta e$, increases in a bigger wealth difference between the auditors $D - W_l$ ($D$ increases, $W_l$ decreases).

While the reputation effect is ambiguous, the wealth effect clearly benefits the Big 4 auditor. A difference in wealth between the auditors directly translates into a gap in their incentives. Smaller litigation consequences discourages the smaller auditor to choose a high level of audit quality. Hence, the difference in audit quality increases with a stronger wealth restriction on the non-Big 4 auditor, $\frac{d\Delta e}{dW_l} < 0$, or higher litigation damages $\frac{d\Delta e}{dD} > 0$, and the Big 4 auditor provides the higher quality more often. Therefore, litigation plays an important role in explaining differences between
auditor. Not only the wealth difference benefits the Big 4 auditors, even without wealth difference, litigation incentives crowd out reputation incentives as stated in the discussion of Proposition 1. These findings lead to an interesting observation. Regulators around the world currently focus on increasing competition in the audit market and suggest a variety of measures to achieve this goal, such as joint audits or mandatory audit firm rotation. Our model shows that reducing litigation concerns could potentially benefit non-Big 4 auditors since the reputational concerns become more important. This measure can encourage clients to hire smaller auditors because they are better able to compete with the Big 4. The German audit market, which has a limited liability for auditors and higher hurdles for investors lawsuits, could be an indication of this effect. In 2016, Big 4 audit firms in Germany were responsible for only 51% of the public company audit market compared to 61% in the entire EU (Audit Analytics 2016). However, limited liability does not necessarily increase overall audit quality in the market since a reduction in litigation generally reduces incentives for all auditors, given that they are able to fully pay the claims. In line with our result, Khurana and Raman (2004) and Maijoor and Vanstraelen (2006) find that Big 4 auditors are associated with a higher audit quality in the US, which is generally known for its strong litigation regime, but not in Australia, Canada or European countries, which are all considered as lower litigation countries.

In general, the results of our model offer an alternative explanation for the mixed evidence on Big 4 vs non-Big 4 audit quality. Reputation incentives can work (but not always do) in favor of the non-Big 4 auditor and can even compensate the possible disadvantage from the wealth difference. This finding addresses empirical studies based on Nelson et al. (1988) and DeAngelo (1981), which simply assume that Big 4 audit firms always provide higher audit quality (e.g. Becker et al. 1998; Choi and
Wong 2007; and Behn et al. 2008). In Nelson et al. (1988), the auditors differ only in wealth and efficiency. Based on these assumptions, it is not surprising that the Big 4 auditor would also always provide the higher audit quality. Similarly, DeAngelo (1981) suggests that big auditors have stronger reputation incentives because they have 'more to lose'.

4 The Company’s Cost of Capital

Investors receive a profit share from the company in case of a successful project, which we label the cost of capital, because the equity provided by the investors is the only outside funding in this model. Since we assume a competitive capital market, there are no rents to investors, which means that there will always be an investor who participates in the contract with a reservation utility of zero. The investors utility for each auditor $i \in \{l, h\}$ is

$$U_i = p(\alpha_i \Pi - I) + (1 - p)(1 - e_i)(\min\{W_i, D\} - I) = 0$$

Solving for $\alpha_i$ results in

$$\alpha_i^* = \frac{I \Pi}{\Pi} + \frac{(1 - p)(1 - e_i^*)(I - \min\{W_i, D\})}{p \Pi}. \quad (8)$$

Again, we first analyze the case where the wealth difference does not matter, $D < W_i < W_h$, and the auditors only differ in reputation.

**Proposition 2** There is a threshold $\hat{r}_e$ such that a higher ex ante reputation, $r_i$

(i) increases the equilibrium cost of capital, $\alpha_i^*$, for all $r_i < \hat{r}_e$, and 

(ii) decreases the equilibrium cost of capital, $\alpha_i^*$, for all $r_i > \hat{r}_e$. 

From equation (8) it becomes apparent that reputation affects the cost of capital only indirectly via the optimal audit quality, $e^*_i$. A higher audit quality corresponds exactly to a lower cost of capital for the company, \( \frac{d\alpha^*_i}{de^*_i} < 0 \). Therefore, the two opposing forces of reputation from Proposition 1, the incentive effect and the direct cost effect, also impact the cost of capital, but with opposite signs. For a lower ex-ante reputation, \( r_i < \hat{r}_e \), the incentive effect dominates and an increase in \( r_i \) raises the cost of capital, \( \alpha_i \), and for a higher ex-ante reputation, \( r_i > \hat{r}_e \), the cost effect dominates and an increase in \( r_i \) lowers the cost of capital, \( \alpha_i \).

Including the wealth difference, i.e. \( W_l < D < W_h \), the respective cost of capital are

\[
\alpha^*_h = \frac{I}{\Pi} + \frac{(1 - p)(1 - e^*_h)(I - D)}{p\Pi} \quad \text{and} \quad \alpha^*_l = \frac{I}{\Pi} + \frac{(1 - p)(1 - e^*_l)(I - W_l)}{p\Pi}.
\]

with \( \Delta\alpha = \alpha^*_h - \alpha^*_l = \frac{(1 - p) \left[ (1 - e^*_h)(I - D) - (1 - e^*_l)(I - W_l) \right]}{p\Pi} \) \( (9) \)

If \( \Delta\alpha < 0 \), the Big 4 auditor provides the lower cost of capital to the company.

The first derivative with respect to \( W_l \) is given by

\[
\frac{d\Delta\alpha}{dW_l} = \frac{(1 - p)}{p\Pi} \left( \frac{de^*_l}{dW_l} (I - W_l) + (1 - e^*_l)W_l \right) > 0
\]

The results are summarized in the following lemma:

**Lemma 2** If the wealth effect matters, \( W_l < D < W_h \), the difference in cost of capital, \( \Delta\alpha \), decrease in a stronger wealth restriction on the low reputation auditor \( (W_l \text{ decreases}) \)

The auditor has an insurance function, initiated by the transfer of the litigation damage payment, \( D \), to protect the investors from failing projects. If the wealth
difference does not matter, \( D < W_l < W_h \), the size of the damage payment is the same for both auditor and the sign of \( \Delta \alpha \) is determined by the relative reputation of the two auditors. The reputation only enters the equation indirectly via the optimal audit quality, \( e_i^* \). Therefore, the auditor who provides the higher audit quality also provides the lower cost of capital. If the wealth effect matters, \( W_l < D < W_h \), the non-Big 4 auditor has a double-sided handicap. She is discouraged to choose a higher audit quality \( \frac{de_i^*}{dW_l} < 0 \) and in addition not able to provide the investors with the same insurance as the bigger audit competitor. As a consequence, \( \Delta \alpha \) decreases with a stronger wealth restriction on the non Big 4 auditor, \( \frac{d\Delta \alpha}{dW_l} > 0 \), which further benefits the Big 4 auditor.

5 Audit Fee

5.1 Audit Compensation at Net Costs

Before we analyze the total audit fee, we determine each auditor’s compensation, \( AC_i \), when her net costs are exactly covered and she is held at the participation constraint with zero rents, \( U_i^A = 0 \). From (4) we know that this is true for the following audit compensation

\[
AC_i = \frac{k}{2r_i}e_i^{*2} - (1 - p)e_i^*R_i^+ + (1 - p)(1 - e_i^*)(\min\{W_i, D\} + R_i^-). \tag{10}
\]

First, we analyze the effects from reputation without considering the wealth effect, \( D < W_l < W_h \). The first derivative of the audit compensation at net costs is given
by
\[
\frac{dAC_i}{dr_i} = -\frac{k}{2r_i^2}(e_i^*)^2 - (1-p)e_i^*dR_i^+ + (1-p)(1-e_i^*)dR_i^- < 0
\]

The results are summarized in the following proposition.

**Proposition 3** The auditor’s net costs, $AC_i$, decrease in ex ante reputation, $r_i$.

An increase in ex ante reputation affects $AC_i$ in three ways: 1) Similar to (6), there is a direct cost effect from an increase in ex ante reputation. This increase is associated with a better efficiency of the auditor (lower costs per unit audit effort) and the audit compensation at costs decreases as a consequence, $-\frac{k}{2r_i^2}(e_i^*)^2$. 2) A part of the audit compensation can be substituted with the auditor’s incentive to gain reputation. As the ex ante reputation increases, the potential reputation gain becomes smaller, $\frac{dR_i^+}{dr_i} < 0$, which in turn substitutes for less of the audit compensation. Therefore, the auditor’s net costs increase in ex ante reputation, $-(1-p)e_i^*\frac{dR_i^+}{dr_i}$. 3) Contrary to reputation gains, a potential reputation loss has to be covered by the audit compensation at costs. An increase in ex ante reputation reduces the magnitude of the potential reputation loss, $\frac{dR_i^-}{dr_i} < 0$. Hence, the auditor’s net costs decrease in higher ex ante reputation, $(1-p)(1-e_i^*)\frac{dR_i^-}{dr_i}$. Note that the indirect effect via a marginal change in audit effort, $e_i$, does not play a role since it is determined by the first-order condition, $\frac{dAC_i}{de_i} = 0$ (envelope theorem, compare equation (5)). Thus, we only need to consider the direct effects. In total, this means that the compensation necessary to cover the auditor’s net costs sinks when the ex ante reputation increases, $\frac{dAC_i}{dr_i} < 0$. 1) The direct cost effect prevails especially for high ex ante reputation, $r$, since the audit costs are reduced and 3) the reputation loss effect prevails especially for low ex ante reputation, $r$. In sum, these two negative effects always dominate 2) the reputation gain effect.
Including the wealth difference, i.e. $W_l < D < W_h$, the respective audit compensation at net costs for each auditor is

$$AC_h = \frac{k}{2r_h}e_h^*e_h^2 - (1-p)e_h^*(D + R_h^-)$$
$$AC_i = \frac{k}{2r_i}e_i^*e_i^2 - (1-p)e_i^*R_i^+ + (1-p)(1-e_i^*)(W_i + R_i^-).$$

with $\Delta AC = AC_h - AC_i$

$$\Delta AC = (1-p)[D - W_l - 0.5e_h^*(D + R_h^+ + R_h^-) + 0.5e_i^*(W_i + R_i^+ + R_i^-)]$$

The first derivative with respect to the non-Big 4 auditor’s wealth restriction is

$$\frac{d\Delta AC}{dW_l} = -(1-p)(1-e_i^*) < 0$$

**Lemma 3** If the wealth effect matters, $W_l < D < W_h$, the difference in audit costs, $\Delta AC$, increases in a stronger wealth restriction on the low reputation auditor ($W_l$ decreases)

The wealth effect, $W_l < D < W_h$, introduces a new force opposing the effect from reputation. The threat of litigation decreases for the low reputation auditor with decreasing wealth because she is not able to cover the full litigation payment to the investors. Consequently, the auditor demands a lower compensation from the client company for the expected risk, which decreases the audit costs, $\frac{dAC_i}{dW_l} > 0$. As before, the indirect effect via a marginal change in audit effort, $e_i^*$, has no effect on the audit costs since it is determined by the first-order condition, $\frac{dAC_i}{de_i} = 0$ (envelope theorem, compare equation (5)). Hence, the wealth difference with respect to the auditor’s net costs benefits the non-Big 4 auditor, $\frac{d\Delta AC}{dW_l} < 0$. 

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5.2 Audit Rents

To sum up the analysis so far: the effect of ex ante reputation on audit quality is ambiguous because reputation incentives are stronger for non-Big 4 auditors but Big 4 auditors are more efficient. This also leads in total to lower net audit costs for the bigger auditor. In addition, the auditor can be separated by wealth, $W_l < D < W_h$. More wealth means stronger incentives from expected litigation to provide a high audit quality, a better insurance function for investors, but a more costly audit. Due to the various counteracting forces it is far from obvious which auditor should be hired from the company’s perspective.

For this reason, we look next at the company’s utility for each auditor $i \in \{l, h\}$ which is given by

$$U_i^C = p(1 - \alpha_i^*)\Pi - F_i^*.$$ 

The company receives a cash flow, $\Pi$, only if the project is successful and has to give up share $\alpha_i^*$ of this cash flow to the investors in exchange for the project funding. Further, it has to pay audit fee, $F_i^*$, to the hired auditor. So far, we only determined the auditor’s compensation at net costs. In case of perfect competition on the audit market, the auditor would earn zero rents and it is true that $F_i^* = AC_i^*$. As a first step, we assume zero rents for each auditor to analyze, how the auditor’s ex ante reputation affects the cost/benefit trade off of the client firm.

Suppose, the wealth difference does not matter, $D < W_l < W_h$. The auditor’s ex ante reputation affects the company’s utility via the cost of capital, $\alpha_i^*$, and the audit net cost, $AC_i$. Therefore, the total effect is the sum of Proposition 3 and 2. When considering only the audit net costs, the company will always prefer the auditor
with the highest reputation possible, because the audit net costs are decreasing in reputation due to higher efficiency of the Big 4 auditor, $\frac{dAC_i}{dr_i} < 0$. However, the cost of capital effect is ambiguous and increases (decreases) for $r_i < \hat{r}_e$ ($r_i > \hat{r}_e$) due to the U-shaped influence of reputation on audit quality. The overall effect is in the following proposition.

**Proposition 4**  
(i) If the audit is not very effective, $k > \hat{k}_C$, the utility of the company, $U_i^C$, always increases with an increase in the auditor’s ex ante reputation, $r_i$.

(ii) If the audit is effective, $k < \hat{k}_C$, the utility of the company, $U_i^C$ decreases when the ex ante reputation increases for all $r_i < \hat{r}_C$ and increases when the ex ante reputation increases for all $r_i > \hat{r}_C$.

Proposition 4 states that the utility of the company does not necessarily increase when engaging an audit firm with higher reputation. The auditor’s ex ante reputation affects the company’s utility via the cost of capital, $\alpha_i^*$, and the audit net cost, $AC_i$. Therefore, the total effect is the sum of Proposition 3 and 2. Depending on the parameter constellation, one of the two factor dominates the other. We pin down the trade off on the general effectiveness of an audit, $k$, which can depend, for example, on the industry of the client or the audit market’s best practices. If the audit is not very effective, $k > \hat{k}_C$, the company’s priority is on saving costs on the audit, $AC_i$. Therefore, it would choose an auditor with a high ex ante reputation because it has lower marginal costs of performing the audit, $\frac{dAC_i}{dr_i} < 0$. On the contrary, if the audit is effective $k < \hat{k}_C$, then the auditors are better able to provide high audit quality, which reduces the company’s cost of capital, $\alpha_i^*$. As established before in Proposition 1, the effect of ex ante reputation on audit quality is U-shaped ($\frac{d\epsilon^*}{dr_i} < 0$ for all $r_i < \hat{r}_e$
and \( \frac{de^*_i}{dr_i} > 0 \) for all \( r_i > \hat{r}_e \). This means that, contrary to conventional wisdom, the client’s firm value can decrease in the auditor’s ex ante reputation. However, while the smaller auditor can produce a higher audit quality for all \( r_i < \hat{r}_e \), she can provide a higher firm value only for all \( r_i < \hat{r}_C \) with \( \hat{r}_C < \hat{r}_e \). The reason for the smaller range is that the cost advantage of having a bigger ex ante reputation always counteracts the higher incentives from reputation loss of the smaller auditor. In other words, higher audit quality does not always coincide with higher firm value for the client because the costs can raise in a faster rate than the benefits.

Next, we lift the assumption of zero rents for the auditors. Our model comprises two auditors with different characteristics who compete on cost and quality, which means that we do not have perfect competition. The auditor who provides the higher firm value to the client is in the position to decline an offer. This position naturally shifts at least a fraction of the bargaining power to her and allows her to grasp a fraction of the benefit to the firm over its next best option, e.g. the audit competitor. Consequently, the audit rent for the better auditor is

\[
AR_{hl} = \lambda * \Delta U^C_{hl} = \lambda * [U^C_h(F^*_h = AC_h) - U^C_l(F^*_l = AC_l)]
\]

(12)

\[
= \lambda (-\Delta \alpha * ps \Pi - \Delta AC)
\]

where the fraction \( \lambda (1 - \lambda) \) describes the auditor’s (firm’s) exogenous bargaining power.\(^{\text{10}}\)

If the audit rent, \( AR_{hl} \), is positive (negative), it means that the Big 4 (non-Big 4) auditor provides a benefit to the company, which is added to the audit compensation

\(^{\text{10}}\)From a purely economic perspective, our model should render \( \lambda = 1 \). However, many other factors are likely to play a role, which is why we allow a range of possible constellations, \( \lambda \in [0,1] \).
at net costs and together forms the equilibrium audit fee.

\[ F^* = AC_h + \max\{AR_{hl}; 0\} \quad \text{and} \quad F^*_l = AC_l - \min\{AR_{hl}; 0\}. \]

For the company, it is always beneficial to pick the auditor with the audit rent because it also implies a benefit for the company itself. The utilities of the company with the high (low) reputation auditor can be displayed as

\[ U_C^h = U_C^l(F^*_l = AC_l) + \max\{(1 - \lambda)\Delta U_C^h; 0\} \quad \text{and} \quad U_C^l = U_C^h(F^*_h = AC_h) - \min\{(1 - \lambda)\Delta U_C^l; 0\}. \]

Without the wealth effect, \( D < W_l < W_h \), the auditors only differ in ex ante reputation. This means that Proposition 4 about the effect of ex ante reputation on the firm value can be applied one-to-one to the audit rents, \( AR_{hl} \). Proposition 4 claims that both the high and the low reputation auditor can potentially contribute to a higher firm value due to the U-shaped characteristic of the reputation effect on the company’s utility. This benefit is shared between the company and the auditor according to the relative bargaining power, \( \lambda \), and allows the auditor to grasp positive rents. Therefore, it depends on the specific constellation, foremost the audit effectiveness in the market \( k \) and the relative distance between the auditor’s ex ante reputation \( r_h - r_l \), which auditor provides the bigger benefit to the company. It is also possible that both provide the exact same benefit, \( AR_{hl} = 0 \) with \( r_h > r_l \).

Next, we discuss the influence of the wealth effect, \( W_l < D < W_h \).

**Proposition 5** *If the wealth effect matters, \( W_l < D < W_h \), the audit rent \( AR_{hl} > 0 \) (\( AR_{hl} < 0 \)) increases (decreases) for the Big 4 (non Big 4) auditor with a stronger wealth restriction on the non-Big 4 auditor (\( W_l \) decreases).*
The low reputation auditor’s wealth influences the audit rent in line with its two main components, the difference in investors’ profit share, $\Delta \alpha$, and the difference in audit costs, $\Delta AC$: The wealth effect benefits the Big 4 auditor with respect to 1a) audit quality (Lemma 1) and 1b) insurance function (Lemma 2). Both effects 1a) and 1b) result in an increased difference in cost of capital. However, 2) the wealth effect decreases also the non-Big 4 auditor’s net costs, $\Delta AC$, and therefore, makes her cheaper compared to the Big 4 auditor (Lemma 3). In total, effects 1a) and 1b) always dominate effect 2). Therefore, the wealth effect benefits the Big 4 auditor and the Big 4 (non-Big 4) auditor’s rent always increases (decreases) with a wealth constraint on the low reputation auditor ($\frac{dAR_{hl}}{dW_l} > 0$).

We further entertain the possibility that only the Big 4 auditor is capable of providing audit services to the client, i.e. if the client is too big or too complex to be audited by a firm with a medium or low reputation, $r_l < \bar{r}(s) < r_h$. This analysis is a special case of the two auditors scenario, where $e^*_l = 0$ and $R_{l}^{-}$ is not defined and can also be set to zero. Consequently, the high reputation auditor is no longer compared to her competitor but to the case where the firm does not hire an auditor at all. The audit rent changes to

$$AR_h = \lambda [U^C_h (F^*_h = AC_h) - U^C_l (R^{-}_l, R^*_l, e^*_l = 0)]$$

**Proposition 6** The Big 4 auditor’s rent is higher when she is the only auditor capable of providing audit services to the client, $AR_h > AR_{hl}$.

Proposition 6 confirms a trivial result. If the off-equilibrium option becomes less attractive (non-Big 4 auditor compared to no other auditor), the Big 4 auditor’s rent increases assuming that she is still able to provide a benefit to the company which exceeds the costs of auditing. The bigger the client, the smaller the selection of
auditors which are capable of providing the resources for an adequate audit. Consequently, some large conglomerates can, indeed, only pick a Big 4 auditor. These rents are potentially the source of the dominance of Big 4 auditors and their competitive advantage over smaller auditors, not their higher reputation per se.

5.3 Big 4 Audit Premium

One of the biggest trends in empirical auditing literature in the past decades was to find out if client companies are willing to pay a fee premium to acquire the service of a Big 4 auditor. Thereby, one of the main problem is the missing data on the auditor’s cost structure which is mostly proprietary information. Therefore, most studies use audit fees as a proxy for audit quality and assume that the cost structure of all auditors must be constant. However, the identification of the underlying drivers for a possible premium in audit fees is still a remaining challenge. If there is a Big 4 premium, what does it tell us? In order to approach this question, we decomposit the difference in audit fees between the two auditors in this model.

\[ AP_{B4} = F_h^* - F_l^* = \Delta AC + AR_{hl} \]

\[ = (1 - \lambda)\Delta AC - \lambda \Delta \alpha * ps\Pi \]

Equation (13) shows the two components of a possible Big 4 premium according to the relative bargaining power of the client and the auditor, a difference in net audit costs, \( \Delta AC \), and a difference in cost of capital, \( \Delta \alpha \). Let’s first consider two extreme scenarios. If all the bargaining power is completely with the company, \( \lambda = 0 \), the audit premium is purely cost based, \( \Delta AC \). As mentioned before in Proposition 3, the audit net costs decrease with an increase in reputation, which means that the Big 4 auditor has a cost advantage over the non-Big 4 auditor and \( \Delta AC \) is negative. Thus, there
can only be an audit premium if the wealth difference between the auditors matters and the difference is big enough, \( W_l \ll D \), which turns \( \Delta AC \) positive. Otherwise, the non-Big 4 auditor earns a fee premium. However, this fee premium is not necessarily connected to a higher audit quality.

Contrary, if all the bargaining power is with the auditor, \( \lambda = 1 \), the audit premium is purely based on the cost of capital, \( -\Delta \alpha \cdot ps\Pi \). The reason is that the benefit to the company from hiring the auditor, who provides the higher firm value, falls entirely to the auditor in form of the audit rent, \( AR_{hl} \). The audit rent partially includes the cost (dis-)advantage of the auditors (compare equation (12)), which entirely offsets the cost component of the Big 4 premium, if \( \lambda = 1 \). Consequently, the Big 4 auditor earns an audit premium, whenever it can provide a lower cost of capital, \( \Delta \alpha < 0 \). In this case, the audit premium is solely based on audit quality. Therefore, we can already state that the Big 4 premium allows for different conclusions depending on the parameter constellation and is only an indicator for higher audit quality if the major part of the bargaining power is indeed with the auditor. We conduct further comparative statics in the following proposition:

**Proposition 7**  
(i) If the the wealth effect does matter, \( W_l < D < W_h \), the Big 4 premium, \( AP_{B4} \), always increases with the damage payment, \( D \).

(ii) If the wealth difference does not matter, \( D < W_l < W_h \), the Big 4 premium, \( AP_{B4} \), decreases (increases) in the damage payment, \( D \) if \( e_l^* < e_h^* (e_l^* > e_h^*) \).

Proposition 7 shows that the effect of the damage payment, \( D \), on the Big 4 premium is ambiguous. The damage payment affects both components of the Big 4 premium: 1) the difference in audit costs, \( \Delta AC \), and 2) the difference in cost of capital, \( \Delta \alpha \). If the non-Big 4 auditor cannot serve the litigation claim from an
investors’ lawsuit, $W_l < D < W_h$, then only the Big 4 auditor is affected by a change in the damage payment, $D$. It is intuitive that the net audit costs increase in the damage payment $\frac{dAC_h}{dD} > 0$ since the expected litigation threat increases and the auditor is incentivized to induce more audit quality. Similarly, the cost of capital decrease due to the higher audit quality and the better insurance function of the auditor $\frac{d\alpha^*_h}{dD} < 0$. Both effects result in an increase of the Big 4 premium, $\frac{dAP_{B4}}{dD} > 0$.

When the wealth difference does not matter, $D < W_l < W_h$, both auditors are affected by a change in the damage payment, $D$. Interestingly, the overall effect only depends on which auditor provides the higher audit quality. When the Big 4 auditor provides the higher audit quality, $e_l^* < e_h^*$, then the difference in audit costs decrease, $\frac{d\Delta AC}{dD} < 0$, and the difference in costs of capital increase, $\frac{d\Delta \alpha}{dD} > 0$. Both effects act in the same direction and the positive Big 4 premium decreases in the damage payment, $D$. The effect exactly reverses, if the non-Big 4 auditor provides the higher audit quality, $e_l^* > e_h^*$.

Similar to this result, Choi et al. (2008) find in an empirical study that the Big 4 premium decreases in the strictness of the liability regime. They derive this hypothesis based on an analytical model which is differently specified than ours. Most prominently (but not exclusively), they understand the strictness of the litigation as the probability of a damage payment and the auditors only differ in wealth. In Proposition 7 (ii), we show that the same result can also be achieved with an increase in the magnitude of the damage payment and an audit quality effect which roots in the difference in reputation between the auditors. This comparison is only one example of how we contribute to the audit fee literature. We provide a rich structure of underlying effects which could potentially explain the vast amount of mixed empirical findings.
6 Conclusion

In this paper, we investigate analytically the role of reputational concerns in auditing. Our understanding of reputation is motivated by Bayesian belief updating, in the sense that the current reputation of an auditor represents her history of successful and unsuccessful signals. Therefore, the uncertainty about the type of the auditor is higher for lower reputation auditors and an additional signal affects more strongly the auditor’s reputation. In line with Bayesian belief updating, we assume that reputation is a concave function, which implies that the potential reputation gain and loss decreases in current reputation. We show that both audit quality and the firm value of a client are U-shaped in the auditor’s current reputation. Hence, higher reputation (larger) auditors do not necessarily provide an higher audit quality and are therefore not necessarily preferred by the company. These results hold, despite taking into account that the audit cost efficiency increases with the auditor’s reputation.

The driving force of these results is the assumption of a concave reputation function. In contrast to DeAngelo (1981), we argue that reputational concerns are more pronounced for auditor’s with lower current reputation, because an additional signal (successful or unsuccessful audit) has a greater impact on beliefs about the type of the auditor. Further, for a fixed client size, the threat of being forced to exit the audit market is higher for auditors with a lower ex ante reputation, because the reputational damage is more likely beyond repairable.

We derive a series of empirical implications from an extension of our model. Therefore, we take also into account that auditors can differ with respect to wealth, whereby the auditor with higher reputation has also higher wealth. The wealth consideration matter when one or both auditors are not able to cover the damage payment from
an investor lawsuit. While these wealth consideration dampens the low reputation auditor’s incentives to provide high audit effort, we still find that high reputation auditor do not necessarily provide a higher audit quality.

Further, our results contribute to a deeper understanding of Big 4 premia. We identify two underlying components of Big 4 premia: 1) the differences in audit costs between Big 4 and non-Big 4 auditors and 2) the differences in costs of capital each auditor can provide. The weighting of the first component decreases with the auditor’s bargaining power relatively to the second component. When the auditor has no (all) bargaining power, then the Big 4 premium consists only of the audit costs (costs of capital) component. Since Big 4 auditors are more efficient in our model, the existence of Big 4 premia is more likely when the auditor holds more bargaining power. Further, we show that Big 4 premia do not correspond one-to-one with audit quality, since the costs of capital component is also affected by the insurance function of the auditor.

Finally, our model is also subject to some limitations. We do not consider independence issues by assuming that the auditor will always report truthfully. However, Shapiro (1983) argues that the option to maintain reputation must be more valuable than the option to mis-use reputation in order to gain short-term profits. One way to view our model is that we normalize the option to mis-use reputation to zero in that we assume that the auditor always reports truthfully and is therefore always independent. Thus, including any benefits from mis-using reputation will not affect our results as long they are always lower than the benefits from maintaining reputation.
Appendix A - Proofs

Proof that $e^*$ is the Equilibrium Audit Effort

The first derivative of equation (4) with respect to $e$ is given by

$$
\frac{d[\ldots]}{de} = \frac{ke}{r} - (1 - p)(D + R^+ + R^-)
$$

(14)

Setting equation (14) equal to zero and rearranging yields

$$
e^* = \frac{(1 - p)r(D + R^+ + R^-)}{k}
$$

The second derivative is given by

$$
\frac{d^2[\ldots]}{de^2} = \frac{k}{r} > 0
$$

which is larger than zero and thus, $e^*$ minimizes the optimization problem (4).

Proof of Proposition 1

Part I - Determining the extremum and the type of the extremum

The first derivative of $e^*$ with respect to $r$ is given by

$$
\frac{de^*}{dr} = \frac{(1 - p)(D + R^+ + R^-)}{k} + \frac{(1 - p)r\left(\frac{dR^+}{dr} + \frac{dR^-}{dr}\right)}{k}
$$

positive effect due to cost reduction

negative effect due to decreased reputation gain and loss

$$
= \frac{(1 - p)\left(D + \sqrt{r + s} - \sqrt{r - s} + \frac{1}{2}r\left(\frac{1}{\sqrt{r + s}} - \frac{1}{\sqrt{r - s}}\right)\right)}{k}
$$

whereby

$$
\frac{dR^+}{dr} = \frac{1}{2}\left(\frac{1}{\sqrt{r + s}} - \frac{1}{\sqrt{r}}\right) < 0
$$

$$
\frac{dR^-}{dr} = \frac{1}{2}\left(\frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r - s}}\right) < 0
$$
Setting $\frac{de^*}{dr}$ equal to zero and rearranging yields the implicit definition of the threshold $\hat{e}_r$ that minimizes $e^*$

$$D = \sqrt{\hat{e}_r - s} - \sqrt{\hat{e}_r + s} - \frac{1}{2}\hat{e}_r \left( \frac{1}{\sqrt{\hat{e}_r + s}} - \frac{1}{\sqrt{\hat{e}_r - s}} \right)$$ (15)

Further, the requirement that the damage payment $D$ must be positive provides an impression where we can expect this extremum. This requirement implies that (15) can only be satisfied for $r < \frac{2}{\sqrt{3}} s$ and thus, any $\hat{e}_r$ resulting form equation (15) must be less than $\frac{2}{\sqrt{3}} s$. This helps by determining the type and uniqueness of the extremum.

$$D = \sqrt{r - s} - \sqrt{r + s} - \frac{1}{2} r \left( \frac{1}{\sqrt{r + s}} - \frac{1}{\sqrt{r - s}} \right) > 0$$

Squaring and rearranging yields

$$(r^2 - \sqrt{r^2 - s^2})(4(r^2 - s^2) - r^2) < 0$$ (16)

$(r^2 - \sqrt{r^2 - s^2}) > 0$ because $s > 0$ and thus,

$$4(r^2 - s^2) - r^2 < 0$$ (17)

must hold in order that (16) is true.

Rearranging equation (17) yields the condition

$$r < \frac{2}{\sqrt{3}} s$$

Next, we take a look at the second derivative of $e^*$ with respect to $r$ which is given by

$$\frac{d^2e^*}{dr^2} = \left(1 - p\right) \left(\frac{1}{4} r \left( \frac{1}{(r-s)^{3/2}} - \frac{1}{(r+s)^{3/2}} \right) - \frac{1}{\sqrt{r-s}} + \frac{1}{\sqrt{r+s}} \right)$$

ks

The second derivative is positive if

$$\frac{1}{4} r \left( \frac{1}{(r-s)^{3/2}} - \frac{1}{(r+s)^{3/2}} \right) - \frac{1}{\sqrt{r-s}} + \frac{1}{\sqrt{r+s}} > 0$$ (18)

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Rearranging yields

\[
\frac{1}{4} \frac{r}{(r-s)^{\frac{3}{2}}} - \frac{1}{\sqrt{r-s}} > \frac{1}{4} \frac{r}{(r+s)^{\frac{3}{2}}} - \frac{1}{\sqrt{r+s}}
\]

\[
\frac{1}{\sqrt{r-s}} \left( \frac{1}{4(r-s)} - 1 \right) > \frac{1}{\sqrt{r+s}} \left( \frac{1}{4(r+s)} - 1 \right)
\]

\[
\frac{1}{\sqrt{r-s}} \left( \frac{4s-3r}{4(r-s)} \right) > \frac{1}{\sqrt{r+s}} \left( \frac{-4s-3r}{4(r+s)} \right)
\]

(19)

Note, that the right-hand side of this inequality is always negative. The sign of the left-hand side depends on the sign of \(4s - 3r\). This term is clearly decreasing in \(r\). Inserting the maximum \(r\) of \(\frac{2}{\sqrt{3}}s\) yields \(4s - 3r = (4 - 2\sqrt{3})s > 0\). Thus, for every possible extremum resulting from equation (15) the inequality (19) holds always. This implies that the second derivative \(\frac{d^2e^*}{dr^2}\) is always positive for all \(\hat{r}_e\). Thus, \(\hat{r}_e\) is a minimum.

Part II - Proof of the existence and uniqueness of the minimum

For fixed exogenous parameters, the requirement \(k > k_{min} = \frac{(1-p)r(D+R^+ + R^-)}{s}\) provides a restriction on the possible values of \(r\). This restriction ensures that the audit effort cannot become higher than 1. Based on the nature of this restriction, the minimum \(\hat{r}_e\) must always exist for all meaningful sets of the exogenous parameters. Otherwise, no value of \(r\) would be able to fulfil the requirement \(k > k_{min}\) and our model would provide no results.

To proof the uniqueness of the minimum \(\hat{r}_e\), we take a look at derivative of (15) with respect to \(r\), which is given by

\[
\frac{1}{\sqrt{r-s}} - \frac{1}{\sqrt{r+s}} + \frac{1}{4} r \left( \frac{1}{(r+s)^{\frac{3}{2}}} - \frac{1}{(r-s)^{\frac{3}{2}}} \right)
\]

Note, that this equals exactly minus the term in (18) which provides the condition when the second derivative, \(\frac{d^2e^*}{dr^2}\), is positive. Since (18) is positive for all possible values of the minimum, \(r < \frac{2}{\sqrt{3}}s\), (6) must be always negative. Hence, (15) is strictly
decreasing in \( r \). This implies that only one value of \( r \) can satisfy condition (15) and thus, the extrema must be unique.

**Proof of Lemma 1**

If the wealth effect matters, \( W_i < D < W_h \), the first derivatives of \( \Delta e \) with respect to \( W_i \) and \( D \) are given by

\[
\frac{d\Delta e}{dW_i} = -\frac{(1 - p)}{k} r_i < 0 \\
\frac{d\Delta e}{dD} = \frac{(1 - p)}{k} r_h > 0
\]

**Proof of Proposition 2**

The first derivative of \( \alpha^* \) with respect to \( r \) is given by

\[
\frac{d\alpha^*}{dr} = -\frac{(1 - p)(I - D)}{p\Pi} \frac{\delta e^*}{\delta r}
\]

The remaining proof is equivalent to the proof of Proposition 1.

**Proof of Lemma 2**

If the wealth effect matters, \( W_i < D < W_h \), the first derivative of \( \Delta \alpha \) is with respect to \( W_i \) given by

\[
\frac{d\Delta \alpha}{dW_i} = \frac{(1 - p)\left( \frac{de^*_i}{dW_i}(I - W_i) + (1 - e^*_i) \right)}{p\Pi} \\
= \frac{(1 - p)\left( \frac{(1 - p)r_i}{k}(I - W_i) + (1 - e^*_i) \right)}{p\Pi} > 0
\]
Proof of Proposition 3

The first derivative of $AC^*$ with respect to $r$ is given by

$$
\frac{dAC^*}{dr} = -\frac{ks}{2r^2}(e^*)^2 - (1 - p)e\frac{1}{2} \left( \frac{1}{\sqrt{r + s}} - \frac{1}{\sqrt{r}} \right)
$$

negative effect due to increased cost efficiency

$$
+ (1 - p) * (1 - e^*) \frac{1}{2} \left( \frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r - s}} \right)
$$

positive effect due to decreased possible reputation gain

$$
+ (1 - p) * (1 - e^*) \frac{1}{2} \left( \frac{1}{\sqrt{r + s}} - \frac{1}{\sqrt{r}} \right)
$$

negative effect due to decreased possible reputation loss

In order to prove that the audit net costs, $AC$, is always decreasing in $r$, it is sufficiently to show that the negative effect of the decreased reputation loss absorbs the positive effect of the decreased reputation gain.

$$(1 - p)(1 - e) \frac{1}{2} \left( \frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r - s}} \right) - (1 - p)e \frac{1}{2} \left( \frac{1}{\sqrt{r + s}} - \frac{1}{\sqrt{r}} \right) < 0$$

rearranging yields

$$e < \frac{1}{\sqrt{r + s} - \sqrt{r - s}}$$

Since $\frac{1}{\sqrt{r + s} - \sqrt{r - s}} > 1$ and $e < 1$, this condition is always meet in our model. Thus, the audit net costs are always decreasing with higher reputation.

To see which effect is the main driver for low and high reputation audit firms, we consider the corresponding limits.
\[
\lim_{r \to s} \frac{dAC^*}{dr} = -\lim_{r \to s} \frac{k}{2r^2} \left( \frac{(1-p)r(D + R^+ + R^-)}{k} \right)^2 \\
- \lim_{r \to s} (1-p) \left( \frac{(1-p)r(D + R^+ + R^-)}{k} \right) \frac{1}{2} \left( \frac{1}{\sqrt{r+s}} - \frac{1}{\sqrt{r}} \right) \\
+ \lim_{r \to s} (1-p) \left( 1 - \frac{(1-p)r(D + R^+ + R^-)}{k} \right) \frac{1}{2} \left( \frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r-s}} \right) \\
= - \frac{(1-p)^2(D + \sqrt{2s})^2}{2k} + \frac{2 - \sqrt{2} (1-p)^2(D + \sqrt{2s})}{4k\sqrt{s}} - \infty \\
= - \infty
\]

Thus, for small \( r \) the negative effect due to a decreased possible reputation loss is dominating.

\[
\lim_{r \to \infty} \frac{dAC^*}{dr} = -\lim_{r \to \infty} \frac{k}{2r^2} \left( \frac{(1-p)r(D + R^+ + R^-)}{k} \right)^2 \\
- \lim_{r \to \infty} (1-p) \left( \frac{(1-p)r(D + R^+ + R^-)}{k} \right) \frac{1}{2} \left( \frac{1}{\sqrt{r+s}} - \frac{1}{\sqrt{r}} \right) \\
+ \lim_{r \to \infty} (1-p) \left( 1 - \frac{(1-p)r(D + R^+ + R^-)}{k} \right) \frac{1}{2} \left( \frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r-s}} \right) \\
= - \frac{(1-p)^2D^2}{2k} + 0 + 0 \\
= - \frac{(1-p)^2D^2}{2k}
\]

For audit firms with very high reputation, the audit net costs decreasing effect due to an increased cost efficiency is the dominant effect.
Proof of Lemma 3

If the wealth effect matters, \( W_l < D < W_h \), the first derivative of \( \Delta AC \) is with respect to \( W_l \) given by

\[
\frac{d\Delta AC}{dW_l} = 0.5 \frac{de^*_l}{dW_l} (W_l + R^{-}_l + R^{+}_l) - (1 - 0.5e_l) \\
= \frac{(1 - p) r (W_l + R^{-}_l + R^{+}_l)}{ks} - 1 \\
= e^+_l - 1 < 0
\]

Proof of Proposition 4

The utility of the company is indirectly affected by \( r \) through the audit net costs and the audit effort that effects the cost of capital.

\[
\frac{dU_C}{dr} = -p \Pi \delta\alpha^* \frac{\delta\alpha^*}{\delta r} - \delta AC^* \frac{\delta AC^*}{\delta r} \\
= (1 - p)(I - D) \frac{\delta e^*}{\delta r} - \frac{\delta AC^*}{\delta r}
\]

Since the audit net costs are always decreasing in \( r \), the utility of the company is always increasing in the second argument. Therefore, a decrease in the utility can result only from the first argument. From Proposition 1 we know, that \( e^* \) is only decreasing for relatively small \( r \) with \( r < \hat{r}_e < \frac{2}{\sqrt{3}} s \) and that always a minimum exists. If a function with a minimum is added to an strictly increasing function, than the result is either increasing or has also a minimum. To distinguish these two cases, we consider the left-side limit when \( r \) approaches \( s \). If the company’s utility is decreasing at the left-side limit, a minimum must exists, because the company’s utility is always increasing at the right-hand side limit and the utility is continuous.
\[
\lim_{r \to s} \frac{dU_C}{dr} = \text{sign} \left[ k - (1 - p) \left( I + \sqrt{2s} \right) \right] \infty
\]

The first derivative with respect to \( r \) approaches always infinity in the limit, but the sign depends on the sign of \( (k - (1 - p) \left( I + \sqrt{2s} \right)) \). Rearranging

\[
\left( k - (1 - p) \left( I + \sqrt{2s} \right) \right) < 0
\]
yields the condition

\[ k < \tilde{k}_C = (1 - p)(\sqrt{2s} + I) \]

Thus, if \( k < \tilde{k}_C \), then there exist a minimum \( \hat{r}_C \) that is implicitly defined by

\[
\frac{dU^*_C}{dr} = 0
\]

**Proof of Proposition 5**

\[
AR_{hl} = (1 - p)\lambda \left[ e^*_h \left( I + \frac{R^+_h + R^-_h - D}{2} \right) - e^*_l \left( I + \frac{R^+_l + R^-_l - W_l}{2} \right) - (R^-_h - R^-_l) \right]
\]

The first derivative w.r.t. \( W_l \) is

\[
\frac{dAR_{hl}}{dW_l} = \lambda \frac{(1 - p)r_l}{sk}(W_l - I) < 0
\]

**Proof of Proposition 7**

The first derivative of the Big 4 premium, \( AP_{B4} \), is given by

\[
\frac{dAP_{B4}}{dD} = (1 - \lambda)\frac{d\Delta AC}{dD} - \lambda ps\Pi \frac{d\Delta \alpha}{dD}
\]

(i) When the difference in wealth matters, \( W_l < D < W_h \), then only the high reputation auditor is affected by an increase in the damage payment, \( D \). The difference in the net disutility of the audit, \( \Delta AC \) is given by \( AC_h - AC_l \). Since
only the high reputation auditor is affected by an the damage payment, \( D \), the first derivative with respect to \( D \) is given by

\[
\frac{d\Delta AC}{dD} = \frac{dAC_h}{dD} - \frac{dAC_l}{dD} = \frac{dAC_h}{dD}
\]

From Lemma ?? part (ii), we know that the audit fee, which equals the net disutility of the audit, increases always with the damage payment, \( \frac{dF_h(F=AC)}{dD} > 0 \). Thus, \( \Delta AC \) increases also with \( D \).

The difference in the company’s cost of capital is given by

\[
\Delta \alpha = \frac{(1 - p)[(1 - e^*_h)(I - D) - (1 - e^*_l)(I - W_i)]}{ps\Pi}
\]

An increase in the damage payment, \( D \), leads to a decrease of \( (1 - e^*_h) \) and \( (I - D) \), because \( \frac{dc}{dD} > 0 \) (see Lemma ?? part (ii)). Thus,

\[
\frac{d\Delta \alpha}{dD} < 0 \quad (20)
\]

Taken together the result in equation (??) and (20), the Big 4 premium increases always with the damage payment, \( D \).

\[
\frac{dAP_{B4}}{dD} > 0
\]

(ii) When the wealth effect does not matter, \( D < W_i < W_h \), then \( \Delta AC \) simplifies to

\[
\Delta AC = (1 - p) [(R^{-}_h - R^{-}_l) + 0.5 * (e^*_i(D + R^+_l + R^{-}_l) - e^*_h(D + R^+_h + R^{-}_h)]
\]

The first derivative with respect to \( D \) is then given by

\[
\frac{d\Delta AC}{dD} = (1 - p)0.5 * (e^*_i - e^*_h) \quad (21)
\]
Thus, the sign of $\frac{d\Delta AC}{dD}$ depends on whether the low or the high reputation auditor is able to provide the higher audit quality. According to Proposition 1, both is possible in our model. Further, $\Delta \alpha$ simplifies to

$$\Delta \alpha = \frac{(1 - p)(I - D)(e_l^* - e_h^*)}{p\Pi}$$

The first derivative with respect to $D$ is given by

$$\frac{d\Delta \alpha}{dD} = -\frac{(1 - p)(e_l^* - e_h^*)}{p\Pi}$$  \hspace{1cm} (22)

As in the case of equation (21), the sign of $\frac{d\Delta \alpha}{dD}$ depends on whether the low or high reputation auditor provides the higher audit quality. However, the sign of equation (22) is always opposite to equation (21). Taken the results together, the Big 4 premium, $AP_{B4}$ increases (decreases) with the damage payment, $D$, if the low reputation auditor provides the higher (lower) audit quality.

$$\frac{dAP_{B4}}{dD} = \begin{cases} > 0, & \text{if } e_l^* > e_h^* \\ < 0, & \text{if } e_l^* < e_h^* \end{cases}$$

**Appendix B - Example Bayesian Belief Updating**

**Property 1: Reputation Gain**

Suppose there are two types of players: a good (high quality) and a bad (low quality) type. The type of the players is not directly observable, but signals are sent. The good type player sends a good signal with probability $p$ and a bad signal with probability $(1 - p)$. Similarly, the bad type player sends a good signal with probability $q$ and a bad signal with probability $(1 - q)$ with $q < p$. Further, for simplicity we assume an uninformative prior which implies that $P(\text{good type}) = P(\text{bad type}) = 0.5$. In this
setting we would interpret reputation as the probability that the player is of the good (high quality) type. Thus, reputation is given by

$$P[\text{type = high quality}|g^n b^m] = \frac{p^n(1-p)^m}{p^n(1-p)^m + q^n(1-q)^m}$$

(23)

where $g^n$ and $b^n$ means that $n$ good signals and $m$ bad signals were observed.

First, we show, that reputation always increases if an additional good signal arrives. The first derivative of equation (23) with respect to $n$ is given by

$$\frac{dP}{dn} = \frac{(1-p)^m p^n (1-q)^m q^n \ln(p) - \ln(q)}{((1-p)^m p^n + (1-q)^m q^n)^2}$$

(24)

Since the term in the denominator is squared, the sign of the derivative depends on the sign of the nominator. The term $(1-p)^m p^n (1-q)^m q^n$ is always positive because $p$ and $q$ are probabilities that lie between 0 and 1. Furthermore, $(\ln(p) - \ln(q))$ is positive because $p > q$. Thus, the first derivative with respect to $n$ is always positive and reputation always increases with the number of good signals.

Second, we show, that the reputation gain is decreasing in the current reputation. A higher current reputation is associated with a larger number of good signals. Thus, we consider the second derivative of equation (23) with respect to $n$.

$$\frac{d^2P}{dn^2} = \frac{(1-q)^m q^n - (1-p)^m p^n)(\ln(p) - \ln(q))^2}{((1-p)^m p^n + (1-q)^m q^n)^3}$$

The sign of the second derivative depends on the sign of $((1-q)^m q^n - (1-p)^m p^n)$. This term equals zero when it is equally likely that the signal combination could steam from a low quality type or from a high quality type. This is the case were the uncertainty about the type of the player is the highest. The second derivative is positive if it is more likely that the signal combination is associated to a low quality type. For these bad reputation players increases the reputation gain with the current reputation. However, in case of good reputation players, the second derivative is negative and the reputation gain increases at a decreasing rate.
Property 2: Reputation Loss

With respect to reputation loses, we consider how Bayesian belief updating behaves in case of an additional bad signal. The first derivative of equation (23) with respect to $m$ is given by:

$$\frac{dP}{dm} = \frac{(1-p)^m p^n (1-q)^m q^n (\ln(1-p) - \ln(1-q))}{((1-p)^m p^n + (1-q)^m q^n)^2}$$ (25)

The first derivative is negative because $p > q$. Thus, reputation decreases in case of a bad signal.

The second derivative describes how this reputation loss depends on the current reputation, whereby a higher reputation in associated with more good signals.\textsuperscript{11}

$$\frac{d^2P}{dmdn} = (1-p)^m p^n (1-q)^m q^n (\ln(p) - \ln(q))^*$$

$$* \frac{((1-q)^m q^n - (1-p)^m p^n)(\ln(1-p) - \ln(1-q))}{((1-p)^m p^n + (1-q)^m q^n)^3}$$

The sign of the second derivative depends inversely on the sign of $((1-q)^m q^n - (1-p)^m p^n)$. As before this sign of this term depends whether the player has already a good (bad) reputation, $(1-q)^m q^n < (>) (1-p)^m p^n$. Thus, the second derivative is positive in case of an player with good reputation. Since the first derivative is negative, a positive second derivative means that the reputation loss decreases with increasing reputation.

\textsuperscript{11}We consider the second derivative with respect to $n$ in order to keep our definition of more reputation constant within this example. When considering the second derivative with respect to $m$, the results would be similar, whereby higher reputation would be defined by lower values of $m$. 
Property 3: Reputation Loss is larger than Reputation Gain

This property that the reputation loss is larger than the reputation gain implies that the absolute value of equation (25) is larger than equation (24):

\[
\frac{|(1-p)^mp^n(1-q)^mq^n(\ln(1-p) - \ln(1-q))|}{((1-p)^mp^n + (1-q)^mq^n)^2} > \frac{(1-p)^mp^n(1-q)^mq^n(\ln(p) - \ln(q))}{((1-p)^mp^n + (1-q)^mq^n)^2}
\]

\[
|\ln(1-p) - \ln(1-q)| > \ln(p) - \ln(q)
\]

\[
-\ln(1-p) + \ln(1-q) > \ln(p) - \ln(q)
\]

\[
\ln(1-q) + \ln(q) > \ln(p) + \ln(1-p)
\]

\[
\ln(q(1-q)) > \ln(p(1-p))
\]

\[
q(1-q) > p(1-p)
\]

\[
\frac{(1-q)/(1-p)}{p/q} > 1
\]

where

\[
\frac{(1-q)/(1-p)}{p/q} = \frac{P[\text{signal = type}|\text{signal = b}]/P[\text{signal ≠ type}|\text{signal = b}]}{P[\text{signal = type}|\text{signal = g}]/P[\text{signal ≠ type}|\text{signal = g}]}
\]

(26)

The last inequality compares the odds that the signal matches the type of the player when a bad signal (numerator) or good signal (denominator) is observed. If this odds ratio is higher than one, then the reputation loss is higher than the reputation gain. This condition on the odds ratio requires that it is more likely that the signal matches the type of the player when the signal is bad. This can be interpreted as that the bad signal is more informative than the good signal.

Connection to the Square Root Reputation Function

The concavity of our square root reputation function implies the following characteristics: 1) the reputation gain is decreasing in reputation, \(\frac{dR^+}{dr} < 0\), 2) the reputation
loss is decreasing in reputation, $\frac{dR^-}{dr} < 0$ and 3) the possible reputation loss is larger than the possible reputation gain, $R^- > R^+$. Properties 1) and 2) are only satisfied when the players (auditors) already have a good reputation. However, it is reasonable to assume that there is no market/demand for an auditor which delivers a negative signal in expectation, i.e. an audit failure. This is also reflected in our assumption that an auditor is forced to exit the market when as soon as she delivers a negative signal in expectation. Then, the properties 1) and 2) are always in accordance with Bayesian belief updating.

From the previous section we know that property 3) is only meet when the bad signal is more informative than the good signal. Given that investor lawsuits against auditors are in practice rather rare, the assumption that these are very strong bad signals seems reasonable.

**Numerical Example**

Next, we use numbers to give a better intuition how the updates affect the auditors. $p = 0.7$ is the probability for auditor A and B to conduct a successful audit. $q = 0.2$ is the probability that a random non-active player on the audit market, who also offers audit services and can be hired from the client, provides a succesfull audit. $R = 10$ is the reputational capital that can be gained or lost from the audit which is the same for all engagements. Auditor A has one successful signal, $n = 1$, and auditor B has two successful signals, $n = 2$, both have no unsuccessfull signals, $m = 0$. The market players update according to the formerly mentioned probabilities

$$P_{nm} = P[\text{type} = \text{high quality}|g^n b^m] = \frac{p^n (1-p)^m}{p^n (1-p)^m + q^n (1-q)^m}$$

The ex ante reputation for both auditors is then $P^A_{10} = 0.78$ and $P^B_{20} = 0.92$. Ex
post it is $P_{20}^A = 0.92$ and $P_{30}^B = 0.97$ in case of success and $P_{11}^A = 0.57$ and $P_{21}^B = 0.82$ in case of audit failure. This affects the utility as follows

$$U^A = -(P_{10}^A - P_{11}^A)R + (P_{20}^A - P_{10}^A)R = -0.7 \text{ and}$$

$$U^B = -(P_{20}^B - P_{21}^B)R + (P_{30}^B - P_{20}^B)R = -0.5.$$

The example assumes that the lost reputation capital is the same for both auditors. The incentive from reputation loss is still stronger for auditor A if auditor B’s additional reputation loss on other clients is

$$-(0.78 - 0.57) \times 10 < -(0.92 - 0.82)R$$

$$R < 21$$

In this example, the total rent auditor B could lose is 20 for a total of two clients, which is less than what is necessary to match auditor A’s incentive from reputation loss.

References


