The Risk-Return Paradox for Strategic Management: Disentangling True and Spurious Effect

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Abstract

The relationship between firms’ performance and risk is a central question for strategic management. A common approach to this issue relates the mean and variance of firms’ returns to each other. Employing this approach, Bowman found the puzzling result of a negative relationship, which he termed the “risk-return paradox”. This paper shows that skewness of individual firms’ return distributions has a considerable spurious effect on the mean-variance relationship as it is usually measured. Once size of this effect is determined, a method is devised to disentangle true and spurious effects. The absolute size of the latter is such that, on average, it explains the larger part of the observed negative relationship. The results might contribute to reconciling mean-variance approaches to risk-return analysis with other, ex ante, approaches.

Key words: mean-variance, risk, risk-return paradox, skewness

JEL classification numbers: C81, G39, M29

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1 Introduction

The concept of risk is central to strategic management. In particular, the relationship between the risk and return of firms is of high importance both to practitioners and scholars. Analysis of this relationship is plentiful, as the review by Rueffli et al. (1999) shows. One important strand of the literature measures risk as the variance of a series of returns on equity, assets, or sales. Employing such a mean-variance approach, Bowman (1980) obtained the puzzling result of a negative relationship between risk and return. Since this finding is at odds with the usual and plausible assumption of risk-averse actors, he termed it the “risk-return paradox.” Subsequent work by other authors confirmed his result.


The present work explores a possible misspecification that, so far, has only been mentioned in side remarks. It is a standard assumption in mean-variance analysis that firms’ return distributions are normal (e.g., Bromiley 1991a, Rueffli and Wiggins 1994). If this assumption is violated in such a way that the return distributions exhibit a non-zero skewness, then empirically measured means and variances are spuriously correlated. Already Bowman (1980, p. 27) mentioned this possibility without, however, exploring it further: “If there is some maximum ROE feasible in an industry, then perhaps most variance is really variance down from this upper bound (asymptote). The larger variance is then automatically associated with a lower mean.” While there may be no such upper bound, it may well be that the distribution of returns is left-skewed. That is, the distribution density is characterized by a long tail to the left, with its mean below the median. Now, if a year’s return happens to lie far out in the left tail of the distribution, then the respective firm’s average return is decreased while its variance goes up. Hence, a negative mean-variance relationship can be observed even if all firms had identical, but left-skewed, return distributions.

In this paper, the effect of skewness on the mean-variance relationship is analyzed. First, the spurious correlation between mean and variance due to skewness is analytically calculated for a single firm or, equivalently, for a sample of firms with identical return distributions. This section largely builds on earlier work (Henkel 2000). Then, it is shown that the overall mean-variance relationship is made up of the “true” mean-variance relationship and each firm’s individual spurious correlation due to skewness. A method of how to

\(^1\)See Kahneman and Tversky (1979).
disentangle spurious and real effects is developed and demonstrated using simulated data. Finally, the theoretical results are applied in an empirical analysis, using the Compustat Industrial Annual Database. From a total of 27 industries, 20 are found to exhibit a negative mean-variance relationship, which is significant for 12 of them. Of those 20 (respectively 12) industries, 17 (respectively 11) show a negative average skewness of firms’ return distribution. This gives a first indication that, in explaining the mean-variance relationship, spurious contributions caused by skewness do play a role. Considering their relative size, it is found that spurious contributions can explain the larger part of the observed negative mean-variance relationship. In some industries, the negative association of mean and variance can be fully explained by distribution skewness. These results are even more pronounced when the analysis is restricted to those firms whose average return lies below the respective industry median.

Ruefli et al. (1999) criticize mean-variance approaches to risk-return analysis on the ground that they are borrowed from financial economics and, as ex post concepts, are not linked to ex ante decision theory which concerns managers. While this criticism certainly applies, it does not seem to justify dismissing mean-variance analysis altogether. As ex post concepts, mean-variance approaches allow insightful checks of other, ex ante, methods that are more genuine to the field of strategic management. As one would prefer results from mean-variance analysis to be consistent with those obtained by other methods, the research presented here contributes to reconciling these different approaches to risk-return analysis in the field of strategic management.

The paper proceeds as follows: Section 2 presents the theoretical analysis of the effect of skewness on the mean-variance relationship. In Section 3, this effect is empirically estimated. Section 4 concludes with a summary and a discussion.

2 Theoretical Model

2.1 Spurious Mean-Variance Correlation for Individual Firms

One of the usual assumptions in mean-variance analysis is that firms’ return distributions are stable over time. If they were not, one would encounter an identification problem (Bromiley 1991a, Ruefli 1991, Ruefli and Wiggins 1994). It would not be clear if, e.g., a low return in one year was an unlucky draw from the same distribution that was relevant in the years before or if it was an average draw from a distribution that altogether had shifted downwards. The assumption of a stable distribution is also made in my analysis, which is, to some degree, justified by the findings of Wiseman and Bromiley (1991). These authors tested the effect of hypothetical time trends on the measure of variance and found a persistent negative
risk-return relationship after also correcting for this effect.

The second simplifying assumption that is usually made is that returns are normally distributed. This assumption is relaxed in my analysis; the influence of higher moments is explicitly taken into account. In particular, distributions may be skewed. Let the return of firm $i$ in period $t$, $t = 1 \ldots T$, be given by the random variable $r_{it}$. The $r_{it}$ are assumed independent for all $i$ and $t$. For a particular firm $i$ and all time periods $t$, the $r_{it}$ are modeled to be identically distributed with expected value $\mu_i$, variance $\sigma_i^2$, and third and fourth central moment $\alpha_i^3$ and $\kappa_i^4$, respectively. The goal of my analysis is to determine the relationship between $\mu_i$ and $\sigma_i^2$ across the sample of firms – the “true” mean-variance relationship.

From firm $i$’s returns $r_{it}$ over the time period $t = 1 \ldots T$, estimates for this firm’s expected return $\mu_i$ as well for its variance of returns, $\sigma_i^2$, can be calculated. The random variables $m_i$ and $s_i^2$ describe the distribution of these estimates:

$$m_i := \frac{1}{T} \sum_{t=1}^{T} r_{it}$$

(1)

$$s_i^2 := \frac{1}{T - 1} \sum_{t=1}^{T} (r_{it} - m_i)^2$$

(2)

These are dependent random variables, the joint distribution of which is induced by the distribution of $r_{it}$.

**Proposition 1.** For the variances, covariance, and correlation of $m_i$ and $s_i^2$, the following holds:

$$\text{Var} [m_i] = \frac{\sigma_i^2}{T}$$

(3)

$$\text{Var} [s_i^2] = \frac{1}{T} \left( \kappa_i^4 - \sigma_i^4 \frac{T - 3}{T - 1} \right)$$

(4)

$$\text{Cov} [m_i, s_i^2] = \frac{\alpha_i^3}{T}$$

(5)

$$\text{Corr} [m_i, s_i^2] = \frac{\alpha_i^3}{\sqrt{\sigma_i^2 \left( \kappa_i^4 - \sigma_i^4 \frac{T - 3}{T - 1} \right)}}$$

(6)

**Proof.** Equation (3) is obvious. A proof of (4) can be found, e.g., in Mood et al. (1974, p. 229). Equation (5) is proved in a similar manner (Henkel 2000). (6) follows from (3), (4), and (5).

These equations can be interpreted as follows. First assume all firms’ return distributions are identical and characterized by the central moments $\sigma_i^2$, $\alpha_i^3$, and $\kappa_i^4$. Then the expected correlation between the estimated values for mean and variance across firms is
given by (6). For a left-skewed distribution, this quantity is negative, even though there is no correlation at all between the underlying true means and variances of the firms’ return distributions; these are all identical, equal to \( \mu_i \) and \( \sigma_i^2 \), respectively. While the possible existence of such a purely spurious “risk-return paradox” is obvious, Proposition 1 provides a means to quantify the expected size of the spurious effect.

### 2.2 Disentangling Spurious and Real Effects

In an empirical sample, mean and variance of firms’ returns will obviously vary across firms; there might be a positive or negative relationship between these quantities. However, this sought-for true relationship between means and variances across firms is, in general, intertwined with the spurious effects from individual firms’ return distributions discussed above. Hence, unless skewness is zero for all firms, the estimates for correlations are, in general, biased. How these effects can be disentangled and unbiased estimates can be obtained is shown below.

Consider \((\mu, \sigma^2)\) as a random two-vector obtained by drawing one firm out of the population. That is, if firm \(i\) is drawn, then the vector contains the true mean \(\mu_i\) and the true variance \(\sigma_i^2\) of firm \(i\)’s return distribution. The distribution of \((\mu, \sigma^2)\) for a population of \(N = 4\) firms is illustrated by the black dots in Figure 1, each denoting a point \((\mu_i, \sigma_i^2)\). Let \(\phi\) denote the distribution density of \((\mu, \sigma^2)\) in \((\hat{\mu}, \hat{\sigma}^2)\)-space. Due to the finite number of firms in the population, \(\phi\) is an atomistic distribution, i.e., given by isolated points.

If a particular firm \(i\) is given, then \(T\) draws of \(r_{it}\) yield, by equations (1) and (2), the random vector \((m_i, s_i^2)\). Let the joint distribution of \((m_i, s_i^2)\) be described by the density function \(f_i\), which is illustrated as the shaded oval area around the point \((\mu_i, \sigma_i^2)\). Note that \(f_i\) is centered around \((\mu_i, \sigma_i^2)\), since this is the expected value of \((m_i, s_i^2)\).

The two-stage process of randomly drawing one firm out of the population and then drawing \(T\) return values for this firm yields a random vector \((m, s^2)\). The distribution density \(f\) of this vector results from averaging the individual firms’ distributions \(f_i\) over the population of all firms: \(f(\hat{\mu}, \hat{\sigma}^2) \equiv \frac{1}{N} \left( \sum_{i=1}^{N} f_i(\hat{\mu}, \hat{\sigma}^2) \right)\). It can also be seen as a convolution of the “true” mean-variance distribution \(\phi\) with the individual firms’ distributions \(f_i\). Figure 1 illustrates this convolution. Due to negative skewness, which is assumed in the illustration, the ovals are downward sloping. The figure visualizes how the convolution of the distribution \(\phi\) of \((\mu, \sigma^2)\) with each firm’s distribution \(f_i\) of \((m_i, s_i^2)\) can lead to an overall negative relationship (the total shaded area) between mean and variance for \((m, s^2)\), even though a positive correlation exists for the distribution \(\phi\) (black dots). The following proposition

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2 More precisely, the convolution that yields the density \(f\) is a convolution of the density \(\phi\) with translated densities \(\tilde{f}_i\), where the translation of \(f_i\) to \(\tilde{f}_i\) is such that the expected value of \(\tilde{f}_i\) is \((0, 0)\).
shows how these effects can be disentangled.

**Proposition 2.** The sought-for true covariance and correlation between the two components of the random vector \((\mu, \sigma^2)\) — that is, the *true* means and variances — can be obtained as follows:

\[
\text{Cov} [\mu, \sigma^2] = \text{Cov} [m, s^2] - \mathbb{E}_\phi [\text{Cov} [m_i, s_i^2]] \tag{7}
\]

\[
\text{Corr} [\mu, \sigma^2] = \frac{\text{Cov} [m, s^2] - \mathbb{E}_\phi [\text{Cov} [m_i, s_i^2]]}{\sqrt{\text{Var} [m] - \mathbb{E}_\phi [\text{Var} [m_i]]}\sqrt{\text{Var} [s^2] - \mathbb{E}_\phi [\text{Var} [s_i^2]]}} \tag{8}
\]

In the above equations, \(\mathbb{E}_\phi [\text{Cov} [m_i, s_i^2]]\) stands, with a slight abuse of notation, for the expected value over \(\phi\) of the individual firms’ covariance between mean and variance of returns (see (5)).

**Proof.** See Appendix.

The first term on the right-hand side of equation (7) is what is commonly measured in mean-variance analysis of returns. This measure is biased; the “true” covariance between means and variances across firms requires a correction. It is obtained by subtracting the average of the individual firms’ spurious covariance (5) — the spurious contribution due to skewness — from the (biased) measured covariance. Similar equations hold for the “true”
variances of $\mu$ and $\sigma^2$ (see Appendix), which together with (7) lead to (8).

Equations (7) and (8) are not only theoretical insights, but can be employed to obtain unbiased estimates of the desired quantities on the left-hand side. Estimates for $\text{Var}[m]$, $\text{Var}[s^2]$, and $\text{Cov}[m, s^2]$ are available; these are the commonly used quantities in mean-variance analysis. Unbiased estimates for the expected values $E_\phi[\ldots]$ can be obtained by calculating the sample means of the quantities given in equations (3), (4), and (5). In the next section, these results will be employed in an empirical analysis.

3 Empirical Analysis

3.1 Simulation

In order to illustrate the interaction between spurious and true contributions, as well as how both can be disentangled, a simulation is employed. This will show how, in a sample of firms with positively correlated true means and variances but individual left-skewed return distributions, the standard measurement of the mean-variance relationship can yield a negative correlation. I consider a simulated sample of 1000 firms. The true means $\mu_i$ of the individual ROE distributions are equally distributed in $[0, 0.1]$; the true variances $\sigma_i^2$ equally distributed in $[0.1, 0.2]$. $\mu_i$ and $\sigma_i^2$ are linearly dependent across the sample, with a positive correlation equal to unity. Their distribution ($\phi$) is visible as the bold line in Figure 2. In each time period $t$, $t = 1 \ldots 10$, firm $i$’s ROE obtains as a draw of the random variable $r_{it}$, which has mean $\mu_i$ and variance $\sigma_i^2$. In order to distinguish random variables from actually observed data (or, in this simulation, actual draws), I denote observed values by capital letters. Hence, firm $i$’s observed ROE in period $t$ is $R_{it}$. For simplicity, I use a (left-skewed) triangular distribution, since this allows an explicit calculation of all required moments. Any other left-skewed distribution would qualitatively yield the same results. For example, the single-period ROE values for the median firm in the simulation, which has $\mu_i = 0.05$ and $\sigma_i^2 = 0.15$, are distributed between -1.05 and 0.6, with the distribution density set at zero at -1.05 and linearly increasing up to 0.6. The covariance between $\mu_i$ and $\sigma_i^2$ across the sample is calculated as 0.000833. Figure 2 shows the result of the simulation, that is, the observed values ($M_i, S_i^2$).

It is clearly visible from the scatter plot that the correlation between observed values of mean and variance is negative. The statistical analysis confirms this observation, yielding a correlation of -0.295 and a covariance of -0.00274. Hence, the spurious effect due to skewness has not only biased downwards covariance and correlation but has, in fact, reversed their signs. The data show a “risk-return paradox” even though the true relationship between mean and variance is positive.
In order to estimate the spurious contribution of distribution skewness using (5) and (7), the third central moment $\alpha_i^3$ is estimated for each firm (see Appendix for a description of this and further estimators). Its average over all firms, divided by the number of periods, yields an estimate of the spurious contribution to the measured covariance, $E_\phi [\text{Cov} [m_i, s_i^2]]$ (see (7)). For the simulation shown in Figure 2, this estimate of the spurious contribution to the covariance between mean and variance is -0.00363. It is larger in absolute value than the observed covariance, -0.00274. Applying (7) yields a corrected value of 0.000892, which comes close to the exact, theoretically calculated covariance between true means and variances of 0.000833. This example demonstrates that the analysis devised in Section 2.2 is indeed suited to disentangle true and spurious contributions to the mean-variance relationship.

The analysis above employed the covariance rather than the correlation of $\mu$ and $\sigma^2$. Since the main interest of this work lies in the sign of the mean-variance relationship and its significance, both are equivalent. However, correlations can be interpreted more easily; (8) in principle allows the corrected correlation to be calculated. In order to achieve this, the second, third, and fourth central moment of the ROE distribution were estimated for each firm (see Appendix for the formulae employed), allowing in turn to estimate $\text{Var} [m_i]$ and $\text{Var} [s_i^2]$ (3, 4). Averaging these over all firms yields estimates for $E_\phi [\text{Var} [m_i]]$ and $E_\phi [\text{Var} [s_i^2]]$, which allow a corrected correlation value to be calculated according to (8). For
the simulation above, the measured correlation is -0.295; the corrected value equals 0.665. The latter does have the correct positive sign, but its relative deviation from the true value, 1, is considerably larger than that of the corrected covariance. The reason for this is the following. If the contribution \( E_\phi [\text{Var}[m_i]] \) of the single firms’ ROE distributions to \( \text{Var}[m] \) in the denominator of (8) accounts for most of \( \text{Var}[m] \), then the estimation error of the difference becomes relatively large. The same logic applies to the other root in (8). In the example shown here, the differences have been overestimated. On the other hand, if they are underestimated, incorrectly large or even non-defined values result for the “corrected” correlation. Hence, while correlation values will also be displayed, the focus in the following analysis lies on the corrected value of covariance.

### 3.2 Data and Results

For the empirical analysis, I employ data from Standard and Poor’s Compustat Industrial Annual Database. The analysis is performed on the level of two-digit Standard Industry Classification (SIC), as also done, e.g., by Bowman (1980) and Fiegenbaum and Thomas (1986, 1988). In order to make the results comparable to those obtained by these and other authors (e.g., Oviatt and Bauerschmidt 1991, Ruefli and Wiggins 1994) I use data from the time period 1970 to 1979.\(^3\) ROE is calculated as “Income before extraordinary items – available for common equity” divided by “Common equity – total”.

In order to restrict the influence of outliers, the one percent of individual ROE observations with extreme values is discarded. This is a sensitive step, since outliers obviously have a strong influence on skewness. Ideally, one would want to keep them in the analysis, but restrict their weight. A convenient way to do this is to employ rank correlation, but this would make it impossible to identify and correct for the effect of skewness. Hence, removing extreme values is the best compromise. Varying the percentage of extreme values that are discarded leaves the results qualitatively unchanged, as long as this percentage does not become too large. Deleting an observation with an extreme ROE value from a firm’s time series strongly changes the observed skewness of this firm’s return distribution. Since skewness is central to my analysis, the respective firms are completely deleted in order to avoid bias; only those firms are kept in the sample for which data on ROE is available for each year in the respective period.

I analyze the full ten-year period 1970 to 1979, since shorter time periods make an

\(^3\)Compared to other decades, this time period has been found to show more and stronger negative associations between mean and variance of firms’ returns (Fiegenbaum and Thomas 1986). My analysis is intentionally restricted to this decade, since the goal of this paper is not to assess the mean-variance relationship in general, but rather to demonstrate how it is influenced by spurious effects and how one can, nonetheless, arrive at an unbiased estimate of it.
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<td>-.01323653</td>
<td>0.3957</td>
<td>-.008323</td>
<td>-.0049134</td>
<td>63 %</td>
<td>-.16384618</td>
<td>-.11265437</td>
</tr>
<tr>
<td>65</td>
<td>10</td>
<td>-.01188745</td>
<td>0.2732</td>
<td>-.0137692</td>
<td>-.00011053</td>
<td>99 %</td>
<td>-.38403916</td>
<td>-.01168921</td>
</tr>
<tr>
<td>73</td>
<td>27</td>
<td>.06963657</td>
<td>0.2250</td>
<td>.05536814</td>
<td>.01426842</td>
<td>80 %</td>
<td>.24146238</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Table 1: Empirical analysis of mean-variance relationship and spurious contributions due to skewness. n.a.: result not defined, since one of the roots in equation (8) took on a negative value.

estimate of higher moments of each firm’s return distribution difficult. Furthermore, firms under the one-digit SIC code 9, “Public Administration,” are taken out because of their non-profit nature. Finally, I keep only those industries for which there are at least ten firms in the sample. This leaves 740 firms in 27 industries.\(^4\) Table 1 displays the results of the empirical analysis.

The column “measured covariance” in Table 1 confirms the familiar result that the

\(^4\)The total number of firms in the initial data set was 1858, over a time-period of ten years. For 5211 single-year observations, no ROE could be calculated because of missing data. From the remaining 13369 observations, 134 were excluded because of extreme ROE values, the same number for the highest and lowest values. Next, all observations were deleted which belonged to incomplete time series, leaving 8950 observations, or 895 firms, in the sample. Excluding industries with less than 10 firms, as well as SIC code 9, finally left 740 firms, as shown in Table 1.
majority of industries (20 out of 27) exhibit a negative mean-variance relationship. Relating the measured covariance to the column “spurious effect” shows a strong correlation: in almost all cases, the entries carry the same sign. Table 2 makes this point precise. Of those 12 industries where the covariance is significantly negative, 11 show a negative spurious contribution. On the other hand, in all of the 5 industries with a significantly positive covariance, the effect from skewness is positive. This result lends empirical support to the hypothesis that distribution skewness has a strong effect on the mean-variance relationship as it is usually measured.

The size of this effect can further be quantified by comparing the relative size of measured covariance and spurious effect, as done in the column “ratio spurious/measured” in Table 1. Averaging the relative size over those industries where the covariance is significantly different from zero yields 75% for the group of industries with negative covariances, 38% for those with positive covariances. While calculating a simple average is a somewhat naive approach, the result nevertheless does strongly suggest that, on average, the larger part of the measured negative mean-variance relationship is in fact spurious, due to negatively skewed return distributions.

The last two columns of Table 1 juxtapose measured and corrected correlations, the latter estimated using (8). For those 20 industries where the measured correlation is negative, the corrected correlation is greater than the measured one in 11 cases (it either becomes positive or the absolute value of the negative correlation decreases), smaller in 6 cases, and not defined in 3 cases. Restricting the analysis to those 12 industries where the negative association is significant, one finds an increased corrected correlation in 6 cases, a decreased one in 5 cases, and an undefined one in one case. This result is less clear-cut than that obtained from the analysis of covariances. However, the corrected correlation values must not be taken at face value. First, as already argued in Section 3.1, estimation errors in the denominator of (8) can translate into large errors of the whole expression. Second, while a directly calculated correlation value can be interpreted as a measure of significance, this is less so for the corrected correlation values calculated by (8). If the correction (7) renders the covariance, which is the nominator in (8), insignificant, then dividing by (possibly small) corrected variance values does not restore significance.

One explanation for the “risk-return paradox” that has been advanced is based on prospect theory (Bowman 1982, Fiegenbaum and Thomas 1988, 1990, Johnson 1992, Sinha 1994). It predicts a negative risk-return relationship for firms with a below-average performance, which indeed has been found empirically (Fiegenbaum and Thomas 1988, Chang and Thomas 1989). This and the results obtained above suggest the need to analyze the effect of distribution skewness separately for under-performing firms in each industry. The result is shown in Table 3. In line with earlier studies, a negative mean-variance relationship is
now found for more industries, 22 out of 27. For 16 (17) of those, it is significant at the 5% (10%) level, as Table 4 illustrates. Calculating, as above, the average contribution of the spurious effect to the measured covariance yields 88%. That is, ROE distributions in the 17 industries with significantly negative covariance are, on average, so strongly negatively skewed that the resulting spurious contribution can largely explain the measured negative relationship between mean and variance. In 8 of these industries, the spurious effect is in fact larger than the measured covariance, such that the corrected covariance turns out to be positive. For reasons discussed above, and since more than half of the values are undefined, the corrected correlation is not further interpreted.

This result suggests an alternative interpretation of the fact that the “risk-return paradox” is more marked for badly performing firms. In all but three industries, the spurious effect is smaller – that is, in most cases, negative and larger in absolute value – in Table 3 than in Table 1. Hence, return distributions for the below-median firms tend to be more negatively skewed than for the industry average. A possible interpretation would thus be that an industry’s low performers are firms with a large negative skewness who had more bad years than others. Hence, the result concerning underperformers could, at least partly, be fallacious and due to the ex post measurement of risk and return: bad years, as long as there are not too many of them, tend to increase the spurious negative relationship between mean and variance, while at the same time leading to a below-average performance.

There is a further effect that higher moments of return distribution can have on the mean-variance relationship (Henkel 2000). Assume firms’ return distributions have a relatively large fourth moment (high kurtosis). That is, return distributions show flat but long tails to both sides. If a year’s ROE value for a certain firm lies in one of these tails, then this firm’s mean ROE is pushed towards the top or the bottom of the population, while its variance goes up. This should result in a U-shaped dependence of empirical variance on the mean. If, in addition, the return distribution is left-skewed, then this U-shape will exhibit a more pronounced left (falling) branch. In fact, this is what Fiegenbaum and Thomas (1988)

<table>
<thead>
<tr>
<th></th>
<th>measured covariance</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>negative</td>
<td>5%</td>
<td>10%</td>
<td>n.s.</td>
<td>5%</td>
</tr>
<tr>
<td>spurious</td>
<td>negative</td>
<td>8</td>
<td>3</td>
<td>6</td>
<td>–</td>
</tr>
<tr>
<td>effect</td>
<td>positive</td>
<td>1</td>
<td>–</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>positive</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Counts of industries in Table 1 by sign of spurious effect and sign of measured covariance (with significance levels)
### Table 3: Empirical analysis of mean-variance relationship and spurious contributions due to skewness, for lower half of firms in each industry w.r.t. mean ROE.

<table>
<thead>
<tr>
<th>SIC</th>
<th>number of firms</th>
<th>measured covariance</th>
<th>p-value</th>
<th>spurious effect</th>
<th>corrected covariance</th>
<th>ratio spurious/measured</th>
<th>measured correlation</th>
<th>corrected correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>-.00032765</td>
<td>0.1145</td>
<td>-0.00036153</td>
<td>-0.0036153</td>
<td>-9 %</td>
<td>-1.5238973</td>
<td>n.a.</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>-.04361294</td>
<td>0.0008</td>
<td>-0.04807042</td>
<td>-0.0445748</td>
<td>110 %</td>
<td>-0.877803</td>
<td>n.a.</td>
</tr>
<tr>
<td>20</td>
<td>19</td>
<td>-.04237403</td>
<td>0.0000</td>
<td>-0.02834413</td>
<td>-0.01402989</td>
<td>67 %</td>
<td>-0.7953284</td>
<td>n.a.</td>
</tr>
<tr>
<td>23</td>
<td>6</td>
<td>-.00159912</td>
<td>0.0431</td>
<td>-0.00844449</td>
<td>-0.0075462</td>
<td>53 %</td>
<td>-1.0558157</td>
<td>n.a.</td>
</tr>
<tr>
<td>25</td>
<td>6</td>
<td>-.00005372</td>
<td>0.3384</td>
<td>-2.452e-06</td>
<td>-0.0005127</td>
<td>5 %</td>
<td>-0.78351139</td>
<td>n.a.</td>
</tr>
<tr>
<td>26</td>
<td>9</td>
<td>-.00672333</td>
<td>0.0001</td>
<td>-0.01200176</td>
<td>.00527844</td>
<td>178 %</td>
<td>-0.9561636</td>
<td>n.a.</td>
</tr>
<tr>
<td>27</td>
<td>11</td>
<td>-.00006987</td>
<td>0.4527</td>
<td>-0.002153</td>
<td>0.0014543</td>
<td>308 %</td>
<td>-2.5312253</td>
<td>13.113843</td>
</tr>
<tr>
<td>28</td>
<td>27</td>
<td>-.00441132</td>
<td>0.9900</td>
<td>-0.0019674</td>
<td>-0.0066896</td>
<td>-3 %</td>
<td>-1.3141789</td>
<td>-1.3141789</td>
</tr>
<tr>
<td>29</td>
<td>7</td>
<td>-.01429731</td>
<td>0.2395</td>
<td>-0.0407263</td>
<td>-0.02649533</td>
<td>285 %</td>
<td>-0.51255185</td>
<td>n.a.</td>
</tr>
<tr>
<td>30</td>
<td>6</td>
<td>-.13127011</td>
<td>0.1378</td>
<td>.04949384</td>
<td>-.18076395</td>
<td>-37 %</td>
<td>-0.67930157</td>
<td>n.a.</td>
</tr>
<tr>
<td>33</td>
<td>12</td>
<td>-.00005499</td>
<td>0.8915</td>
<td>-.0008134</td>
<td>-.00002635</td>
<td>148 %</td>
<td>.0441883</td>
<td>n.a.</td>
</tr>
<tr>
<td>34</td>
<td>16</td>
<td>-.02834053</td>
<td>0.0000</td>
<td>-.01077273</td>
<td>-.01756781</td>
<td>35 %</td>
<td>-1.92771398</td>
<td>n.a.</td>
</tr>
<tr>
<td>35</td>
<td>31</td>
<td>-.01523577</td>
<td>0.0002</td>
<td>-.02040316</td>
<td>-.00516739</td>
<td>134 %</td>
<td>-1.5670553</td>
<td>n.a.</td>
</tr>
<tr>
<td>36</td>
<td>30</td>
<td>-.01421147</td>
<td>0.0002</td>
<td>-.01315999</td>
<td>-.00105148</td>
<td>95 %</td>
<td>-0.62905643</td>
<td>-3.315362</td>
</tr>
<tr>
<td>37</td>
<td>21</td>
<td>-.0053646</td>
<td>0.0091</td>
<td>-.0056627</td>
<td>0.00030286</td>
<td>106 %</td>
<td>-0.5547485</td>
<td>n.a.</td>
</tr>
<tr>
<td>38</td>
<td>13</td>
<td>-.00247396</td>
<td>0.4850</td>
<td>.00665652</td>
<td>-.00418255</td>
<td>269 %</td>
<td>.21288184</td>
<td>n.a.</td>
</tr>
<tr>
<td>48</td>
<td>5</td>
<td>-.0668578</td>
<td>0.0185</td>
<td>-.06917056</td>
<td>-.00231276</td>
<td>103 %</td>
<td>-1.3141789</td>
<td>n.a.</td>
</tr>
<tr>
<td>49</td>
<td>54</td>
<td>-.0004409</td>
<td>0.2333</td>
<td>-.0001022</td>
<td>-.00033387</td>
<td>23 %</td>
<td>-3.0840918</td>
<td>3.2053843</td>
</tr>
<tr>
<td>50</td>
<td>13</td>
<td>-.05953337</td>
<td>0.0000</td>
<td>-.04679727</td>
<td>-.01278365</td>
<td>79 %</td>
<td>-1.2474431</td>
<td>n.a.</td>
</tr>
<tr>
<td>51</td>
<td>8</td>
<td>-.0035376</td>
<td>0.0112</td>
<td>-.0030588</td>
<td>-.00034212</td>
<td>110 %</td>
<td>-1.5670553</td>
<td>n.a.</td>
</tr>
<tr>
<td>53</td>
<td>5</td>
<td>-.0001383</td>
<td>0.6404</td>
<td>.00002974</td>
<td>-.815e-06</td>
<td>164 %</td>
<td>.28638317</td>
<td>-0.65619711</td>
</tr>
<tr>
<td>54</td>
<td>6</td>
<td>-.03447132</td>
<td>0.0326</td>
<td>-.05118663</td>
<td>.01671531</td>
<td>148 %</td>
<td>-1.3141789</td>
<td>n.a.</td>
</tr>
<tr>
<td>58</td>
<td>5</td>
<td>-.0062742</td>
<td>0.8016</td>
<td>.06456013</td>
<td>-.05788592</td>
<td>967 %</td>
<td>.15645724</td>
<td>n.a.</td>
</tr>
<tr>
<td>59</td>
<td>6</td>
<td>-.1633808</td>
<td>0.0001</td>
<td>-.06977397</td>
<td>-.09356411</td>
<td>43 %</td>
<td>-.99003537</td>
<td>-1.0450286</td>
</tr>
<tr>
<td>60</td>
<td>14</td>
<td>-.00451515</td>
<td>0.0007</td>
<td>-.00267488</td>
<td>-.00187027</td>
<td>59 %</td>
<td>-.7927261</td>
<td>-1.8172919</td>
</tr>
<tr>
<td>65</td>
<td>5</td>
<td>-.02137779</td>
<td>0.0640</td>
<td>-.02358176</td>
<td>-.00229397</td>
<td>110 %</td>
<td>-.8561687</td>
<td>n.a.</td>
</tr>
<tr>
<td>73</td>
<td>13</td>
<td>-.14989356</td>
<td>0.0006</td>
<td>-.10428059</td>
<td>-.04561298</td>
<td>70 %</td>
<td>-.82224218</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Table 4: Counts of industries in Table 3 by sign of spurious effect and sign of measured covariance (with significance levels); only firms in lower half of each industry’s performance ranking.
observe. They interpret the U-shape as a confirmation of prospect theory: troubled firms show a high variance because their behavior is risk-loving, while above-average firms benefit from more risky but also more profitable projects. The effect of kurtosis discussed above suggests an alternative interpretation. Just like the “risk-return paradox” itself, the U-shape might, at least partly, be spurious, caused by a high average kurtosis.

The present analysis also has an interesting relation to Oviatt and Bauerschmidt’s work (1991). They compare OLS to 3SLS estimates of the mean-variance relationship. While a negative relationship is found using OLS, it disappears when a 3SLS estimator is employed. The authors conclude that, in their model, both mean and variance of returns are influenced by industry conditions and business strategies, but do not affect each other. The results obtained in the present paper allow to look at Oviatt and Bauerschmidt’s findings from a new angle. OLS estimates are biased when there is a correlation between error terms and exogenous variables; 3SLS allows to remove this bias, given that appropriate instrumental variables are available. In the case of mean and variance of returns, such a correlation is caused by skewness (and, possibly, other factors). Consider a regression analysis with average returns $M_i$ as the dependent variable and empirical variances $S_i^2$ as the explanatory variable. A bad year for firm $i$ then implies a larger negative error term in $M_i$ as well as a higher variance $S_i^2$. Because of skewness, negative outliers are not averaged out by positive ones but instead cause spurious effects. Oviatt and Bauerschmidt also mention, as an aside, a strong correlation between variance and skewness. Their observation suggests what has been shown rigorously here; spurious effects due to skewness constitute an important contribution to the observed mean-variance relationship.

4 Summary and Discussion

Mean-variance analysis is a common approach in assessing the relationship between risk and return. The present paper has pointed out spurious effects in this analysis due to skewness of the individual firms’ return distributions, which distorts the observed relationship. It has been shown how these spurious effects can be taken into account to arrive at a correct estimate of the mean-variance relationship. It turned out that, on average, the spurious effects explain the larger part of the observed “risk-return paradox”.

Of course, the results obtained here hinge on the, widespread and often employed, assumption of an ROE distribution that is stable over time. While it was desirable to relax this assumption, one would run into identification problems (Bromiley 1991a, Rueffli 1991, Rueffli and Wiggins 1994). Furthermore, the main purpose of this paper was to build upon the numerous studies on the relationship between mean and variance which assume stability over time.
Given the strong effect of distribution skewness, the question arises why return distributions should be negatively skewed. Two explanations come to mind. First, a firm’s performance might follow positive swings in external conditions to a lesser degree than it follows negative swings. For example, capacity constraints might make it impossible to take full advantage of an upward shift in demand, while a downward shift has an undampened impact on profits. Second, skewness may be the result of income smoothing. Managers might prefer to accumulate downward deviations from some target level of return, such that a series of relatively stable, high returns would be followed by a rather bad result.

One may generally question the suitability of mean-variance approaches in the context of strategic management, as Ruefli et al. (1999) do. They rightly criticize that it is an ex post concept, which does not reflect the situation of a decision maker. Walls and Dyer (1996), in a study of the petroleum exploration industry, lend empirical support to this criticism, finding that “ex ante risk propensity is not positively associated with the ex post risk measure, variance.” In light of this criticism, I believe the analysis presented here makes a very useful contribution. First, given the widespread attention that mean-variance approaches have received over the last decades, an analysis of the role of skewness seems required. Second, even if one focuses on ex ante concepts, such as the variance of analysts’ profit estimates (Bromiley 1991b) or the content of annual reports (Bowman 1984), one would still prefer the results from mean-variance analysis to be consistent with those from other approaches, as well as with intuition. In fact, correcting for the influence of skewness might make Walls and Dyer’s ex ante and ex post risk measures positively associated. The results presented here should shed some new light on mean-variance approaches to risk-return analysis.
Appendix

Proof of Proposition 2. Let in the following equations ̅µ and ̅σ² denote the expected values of µ and σ². That is, ̅µ :=  \frac{1}{N} \left( \sum_{i=1}^{N} \mu_i \right), and ̅σ² :=  \frac{1}{N} \left( \sum_{i=1}^{N} \sigma_i^2 \right). These are also the expected values of m and s², i.e., ̅µ = m and ̅σ² = s², since the expected value of (mi, σ²i) is (µi, σ²i) for all i. The variables ˆµ and ˆσ² are help variables over which the integrals run. Then Cov [m, s²] can be decomposed as follows:

\[ \text{Cov} [m, s²] = \int \int (\hat{\mu} - \bar{\mu}) (\hat{\sigma}² - \bar{\sigma}²) f(\hat{\mu}, \hat{\sigma}²) d\hat{\mu} d\hat{\sigma}² \] (9)

\[ = \frac{1}{N} \sum_{i=1}^{N} \int \int (\hat{\mu} - \mu_i) (\hat{\sigma}² - \sigma_i²) f_i(\hat{\mu}, \hat{\sigma}²) d\hat{\mu} d\hat{\sigma}² \] (10)

\[ = \frac{1}{N} \sum_{i=1}^{N} \left[ \int \int (\hat{\mu} - \mu_i) (\hat{\sigma}² - \sigma_i²) f_i(\hat{\mu}, \hat{\sigma}²) d\hat{\mu} d\hat{\sigma}² + (\sigma_i² - \bar{\sigma}²) \int \int (\hat{\mu} - \mu_i) f_i(\hat{\mu}, \hat{\sigma}²) d\hat{\mu} d\hat{\sigma}² + (\mu_i - \bar{\mu}) (\sigma_i² - \bar{\sigma}²) \int \int f_i(\hat{\mu}, \hat{\sigma}²) d\hat{\mu} d\hat{\sigma}² \right] \] (11)

\[ = \frac{1}{N} \sum_{i=1}^{N} \text{Cov} [m_i, s²_i] + \text{Cov} [\mu, \sigma²] \] (12)

\[ = E_\phi \left[ \text{Cov} [m_i, s²_i] \right] + \text{Cov} [\mu, \sigma²] \] (13)

The steps from (9) to (10) and (11) are obvious. The first line of (11) equals the first term in (12), which represents the average over the individual firms’ skewness-induced covariance between mean and variance of its returns. The second and third line in (11) vanish, since the expected values of ˆµ and ˆσ² over the distribution fi are µi and σ²i, respectively. The fourth line in (11) is the sought-for true covariance  \frac{1}{N} \left( \sum_{i=1}^{N} (\mu_i - \bar{\mu}) (\sigma_i² - \bar{\sigma}²) \right) between mean and variance of firms’ returns. Note that, since this is a population quantity, not a sample quantity, the division is by the population size N, not N – 1. The last line is merely a reformulation. This completes the proof of equation (7) in Proposition 2.

To prove equation (8), note that

\[ \text{Var} [m] = E_\phi \left[ \text{Var} [m_i] \right] + \text{Var} [\mu] \quad \text{and} \]

\[ \text{Var} [s²] = E_\phi \left[ \text{Var} [s²_i] \right] + \text{Var} [\sigma²] \] (14)

(15)

are proved along the same lines as equation (13). Taking (13) to (15) together yields (8).

Q.E.D.

Estimation of higher moments: Let \( W_i^k \) be defined as the average over all T periods of
the $k$’th power of the deviation of $R_{it}$ from the mean ROE of firm $i$:

$$W_i^k = \frac{1}{T} \sum_{t=1}^{T} \left( R_{it} - \frac{1}{T} \sum_{\tau=1}^{T} R_{i\tau} \right)^k .$$  \hspace{1cm} (16)

Then unbiased estimates for the third and fourth central moment of firm $i$’s return distribution can be expressed in terms of the $W_i^k$ as follows (see Kenney and Keeping 1951, p. 189 for these and the following equations):

$$\hat{\alpha}_i^3 = \frac{T^2}{(T-1)(T-2)} W_i^3 ,$$  \hspace{1cm} (17)

$$\hat{\kappa}_i^4 = \frac{T(T^2 - 2T + 3)W_i^4 - 3(2T - 3)(W_i^2)^2}{(T-1)(T-2)(T-3)} .$$  \hspace{1cm} (18)

The $k$-statistics $k_i^2$ and $k_i^4$ are defined as

$$k_i^2 = \frac{T}{T-1} W_i^2 ,$$  \hspace{1cm} (19)

$$k_i^4 = \frac{T^2 ((T+1)W_i^4 - 3(T-1)(W_i^2)^2)}{(T-1)(T-2)(T-3)} .$$  \hspace{1cm} (20)

Using these expressions, an unbiased estimator of the variance of $s_i^2$, which for known $\sigma_i^2$ is given by (4), is

$$\text{Var} \left[ s_i^2 \right] = \frac{2T(k_i^2)^2 + (T-1)k_i^4}{T(T+1)} .$$  \hspace{1cm} (21)
Industries in the sample\(^5\)

10: Metal Mining
13: Oil and Gas Extraction
20: Food and Kindred Products
23: Apparel and Other Finished Products Made From Fabrics and Similar Materials
25: Furniture and Fixtures
26: Paper and Allied Products
27: Printing, Publishing, and Allied Industries
28: Chemicals and Allied Products
29: Petroleum Refining and Related Industries
30: Rubber and Miscellaneous Plastics Products
33: Primary Metal Industries
34: Fabricated Metal Products, Except Machinery and Transportation Equipment
35: Industrial and Commercial Machinery and Computer Equipment
36: Electronic and Other Electrical Equipment and Components, Except Computer Equipment
37: Transportation Equipment
38: Measuring, Analyzing, and Controlling Instruments; Photographic, Medical and Optical Goods; Watches and Clocks
48: Communications
49: Electric, Gas, and Sanitary Services
50: Wholesale Trade – durable Goods
51: Wholesale Trade – non-durable Goods
53: General Merchandise Stores
54: Food Stores
58: Eating and Drinking Places
59: Miscellaneous Retail
60: Depository Institutions
65: Real Estate
73: Business Services

References


