R&D in a Strategic Delegation Game Revisited

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Summary:
This paper considers a strategic delegation setting with R&D spillovers in a Cournot market. Motivated by the puzzling result in Zhang and Zhang (1997) that delegating the production and R&D decision to managers is never beneficial for the owners of the firm, we reconsider their work to find out possible reasons why delegation can be beneficial. In our analysis the following points are important: (i) The results shown by Zhang and Zhang (1997) are based on the implicit assumption that the owners of the firms cooperate at the contracting stage of the game, where they choose the performance related payment via the incentive parameter in the managerial compensation contracts. This assumption is surprising as all other stages of the game are characterized by noncooperative behavior of the players. In addition this implicit assumption is not in line with the literature on strategic delegation. (ii) For this reason we reformulate the setting and assume that the owners’ behavior is noncooperative at the stage where the compensation contracts of the managers are designed. An additional stage is added at the beginning of the game, where the owners decide whether to delegate or not. (iii) Solving this noncooperative game we find that no closed form solution can be derived. Therefore we characterize the qualitative aspects of the sub-game perfect outcome. (iv) We can show that under certain circumstances delegation is beneficial for the firm owners. Additionally we highlight driving forces in our model that make delegation beneficial and on which further extensions of the model can be build on.

Keywords: Strategic Delegation, Research and Development, Incentives, Strategic Commitment

JEL-Classification: L13, L2, O31
1 Introduction

In this paper we focus on the combination of two separate lines of research dealing with the behavior of firms competing in an imperfect market. The first line, strategic delegation and the effect on market performance, has been started by Vickers (1985) and Fershtman and Judd (1987). They have shown that in a quantity-setting duopoly a firm hiring a manager and compensating this manager on the basis of profits and sales (or, equivalently, on sales revenues), performs better, i.e. has higher profits, than a firm managed by their owners. The reason is that the delegation of the production decision together with writing a contract based on performance measures different than profit works as a self-committing device for the owners and leads to more aggressive behavior of the firm on the market.\footnote{If both owners delegate decisions the opposite holds. This is due to the fact that the firms then find themselves in a situation resembling the well-known prisoner’s dilemma.} The second line, spillovers and the effects on R&D behavior, has been introduced by d’Aspremont and Jaquemin (1988), Suzumura (1992) and Kamien et al. (1992). It has been shown there that industry output and R&D expenditures are higher when firms cooperate on the R&D stage.

Considered separately, the two topics of strategic delegation and R&D spillovers have been the focus of intensive research over the last 15 years. It is somehow surprising that the combination of these two interesting topics has not attracted as much attention, although recently increasingly more work is done in this area; see Kräkel (2003), Lambertini (2004), Zhang (2003). The first who have extended the basic setting were Zhang and Zhang (1997). They study an R&D game with strategic delegation where both, a production decision and an R&D investment decision is made by a manager. R&D expenditures decrease the production costs of the firm, but due to a spillover effect the R&D spending also leads to a decrease in the production costs of the other firm. Their game comprises three stages, a contracting stage, where the owners of the firm design the managerial compensation contracts, the R&D stage, where managers select R&D expenses, and a production stage.

The purpose of the present paper is two-fold. First, we reconsider the work of Zhang and Zhang (1997) and show that the solutions and insights – supposedly derived for the noncooperative case – do not hold. Their results are only valid if one assumes that the owners of the firm cooperate in the contracting stage (i.e. maximize their joint profits), although acting noncooperatively at the production and R&D stage. Since the assumption of cooperation at the contracting stage runs counter to the ideas investigated in the strategic delegation literature, we re-analyze the model and demonstrate that deriving a closed-form solution for this model is not straight-forward. Our second point in this paper is a discussion of ways to characterize the equilibria of the model and the trade-offs between incentives, strategic commitment and spillovers. Furthermore, we present and discuss several extensions. Since the topic of this paper gains increasing attention in the research community, and the fact that research is path-dependent, we believe that it is critical to get the early results in order. This then enables other researchers to build on them.
The paper is organized as follows: In the next section we present the model set-up. In section 3 the analysis of Zhang and Zhang (1997) is revisited in order to point out the difference between the cooperative and noncooperative case in general form. As a closed form solution can not be derived for the noncooperative case, section 4 provides a discussion of the case where discrete values of the incentive parameter and the level of spillovers are assumed. By this it is possible to characterize the equilibria of the game and to illustrate the trade-offs in the model. Section 5 looks at a numerical analysis of the general case. Section 6 provides further extensions of the setting and section 7 concludes.

2 The model

Two firms with homogenous products compete in a market with Cournot competition. Each firm has the same production unit costs of $A$ for their products. These unit costs can be reduced by investing in R&D, where $x_i$ denotes the investment by firm $i$, $i = 1, 2$. Due to spillovers, the R&D spending $x_i$ not only leads to a decrease in the production unit costs for the investing firm $i$ but also to lower unit costs for the rival firm $j$. Let $j$ ($j = 1, 2$ and $i \neq j$) refer to respective rival firm, then firm $i$’s effective production unit costs are $C_i = A - x_i - \theta x_j$, with $\theta \in [0, 1]$ as the measure of the size of the spillover effect. R&D expenses directly diminish the basic production unit costs $A$. There is no uncertainty concerning the effect of the R&D expense on the production unit costs. Investing in R&D is costly however, and the R&D cost function is represented by $\frac{1}{2} r x_i^2$, with $r > 0$. The inverse demand function is given by $p = a - bQ$, with $a > A$ and $Q$ as the industry output: $Q = q_i + q_j$. We consider the following sequential-move game between the two firms.

- Stage 1: Each owner wants to maximize the profit of his firm and has the option to hire a manager to act on his behalf in the R&D and production stage. If he does not hire the manager then the owner determines the R&D level and the production quantity. His payoff in the non-delegation case is

$$\pi_i^{ND} = pq_i - (A - x_i - \theta x_j)q_i - \frac{1}{2} r_i^O x_i^2. \quad (1)$$

If both owners decide to hire managers to run their firms, the profits in this delegation case reads

$$\pi_i^D = pq_i - (A - x_i - \theta x_j)q_i - \frac{1}{2} r_i^M x_i^2 \quad (2)$$

Note that the R&D costs of the agents might be different from the R&D costs of the owners. The owner hires a manager for various reasons: the manager might posses task-specific know-how the owner does not have or

\footnote{For the sake of comparison our model set up follows closely Zhang/Zhang (1997) and is based on identical notations.}
the owner might simply be too busy to carry out the activities for himself. Accordingly, it seems reasonable to assume \( r^M \leq r^O \). Of course, two other branches of the game tree have to be considered, namely the alternatives where only one of the owners decides to delegate the tasks and the other one does not. In following the recent literature on the topic, in this paper we will neglect these cases and just present a comparison of the delegation case and the non-delegation case described above\(^3\).

- **Stage 2**: If the owners decide to delegate the tasks, then a managerial compensation contract is designed by each owner. Based on these contracts all decisions of the following stages of the game, R&D and quantity decision, are delegated to the respective manager. To compensate the managers for his effort, the owner and the manager agree upon a compensation payment determined by the compensation contract. The managerial compensation contracts are based on a linear combination of two performance measures, profit \( \pi \) and sales \( R \).\(^4\) That is, the combined performance measure is given by \( U_i = \alpha_i \pi_i + (1 - \alpha_i) R_i \), with \( \alpha_i \geq 0 \). Profit \( \pi_i \) and sales \( R_i \) are determined by \( \pi_i = pq_i - (A - x_i - \theta x_j)q_i - \frac{1}{2} r^M x_j^2 \) and \( R_i = pq_i \). The actions of the owners in stage 2 are represented by choosing the incentive parameter \( \alpha_i \) noncooperatively, which in turn influences the behavior of the manager at subsequent stages. Observe that \( \alpha_i = 1 \) replicates the profit maximization calculus of the owner managed firm. Choosing \( \alpha_i < 1 \) motivates the manager to act more aggressively in the product market. In order to guarantee that the manager accepts the contract, total compensation must meet the reservation utility requirement. For the analysis it is sufficient to focus on the performance related part of the managerial compensation since the salary is only used for adjusting total compensation to meet the reservation utility requirement.\(^5\)

- **Stage 3**: Given that the components of the compensation contract is determined (i.e. given the values of the incentive parameters \( \alpha_i \) and \( \alpha_j \)), each manager decides upon the R&D expenses \( x_i \) and \( x_j \) in each firm. The R&D expenditures are determined such that the compensation \( U_i \) is maximized.

- **Stage 4**: In the final stage of the game each manager selects the output quantity. Given the values of \( \alpha_i \) and \( \alpha_j \) and the effective production

---

\(^3\)If delegation is beneficial and each owner has access to the market for managers, delegation is each players’ dominant strategy. To analyze the asymmetric case, it becomes necessary to explain why such a situation might occur in the first place; see e.g. Lambertini (2004).

\(^4\)Although we are considering only contracts based on profits and sales, other performance measures might be considered to improve the efficiency. We will discuss this issue later.

\(^5\)This idea can be illustrated by looking at a linear compensation scheme of the following type: \( TC = s_0 + s_i U_i \), with \( TC \) as total compensation, \( s_0 \) representing the salary component and \( s_i \) the bonus rate. Each variation in the amount of the performance based pay can be adjusted by \( s_0 \) resulting in a \( TC \) which exactly meets the reservation utility requirement. See for a detailed discussion e.g. Kräkel (2004).
costs $A - x_i - \theta x_j$, quantities are determined by the managers noncooperatively based on manager’s respective performance measure. The production quantities are determined such that the compensation $U_i$ is maximized.

Note that the presentation of the game slightly differs from the usual approach taken in the literature, e.g. Zhang and Zhang (1997). First, stage 1 is added, where the owners only hire a manager if they can benefit from it. Second, whereas it is usually assumed that R&D costs are the same for the manager and the owner, we allow these costs to be different ($r^M \leq r^O$). Hence, in this sense, our framework is slightly more general. In what follows we will solve this game by backward induction. The solution concept we will use is subgame perfection and we will try to characterize the equilibria of this game.

3 Solving for the optimal strategy

If the owners decide not to delegate, then the equilibrium is symmetric. In equilibrium the owners choose the following quantities and R&D efforts and receive the following profits (if these quantities are greater than zero):

$$q_{1D}^{ND} = q_{2D}^{ND} = \frac{3(a - A)r^O}{9br^O + 2(\theta - 2)(\theta + 1)}$$

$$x_{1D}^{ND} = x_{2D}^{ND} = \frac{2(a - A)(2 - \theta)}{9br^O + 2(\theta - 2)(\theta + 1)}$$

$$\pi_{1D}^{ND} = \pi_{2D}^{ND} = \frac{(a - A)^2 r^O (9br^O - 2(\theta - 2)^2)}{(9br^O + 2(\theta - 2)(\theta + 1))^2}$$

On the other hand, if the owners choose delegation, then we have to solve the corresponding subgame by backwards induction. We start with the production and R&D stage and work ourselves backwards towards the contracting stage, where the optimal value of the incentive parameter is selected.

3.1 The production and the R&D stage

Given that the incentive parameters $\alpha_i$ and $\alpha_j$ have been selected and the R&D expenses $x_i$ and $x_j$ have been chosen, the managers noncooperatively make a decision about the production quantities $q_i$ and $q_j$ such that the performance measure $U_i(\alpha_i, \alpha_j, x_i, x_j, q_i, q_j)$ is maximized. The optimal output of firm $i$ is then given by (again $j$ is the index of the respective rival firm)

$$q_i(\alpha_i, \alpha_j, x_i, x_j) = \frac{a - A(2\alpha_i - \alpha_j) + (2\alpha_i - \alpha_j\theta)x_i + (2\theta\alpha_i - \alpha_j)x_j}{3b}$$

Observe that for $\alpha_i = \alpha_j$ an increase in firm $i$’s R&D expenditures always increases the production quantity due to the cost reducing effect of R&D, whereas

\footnote{See also Kräkel (2003), p. 19ff.}
an increase in the R&D expenditures of firm \( j \) only result in an increase of the production quantity of firm \( i \) when \( \theta > 1/2 \), i.e. when spillovers are sufficiently high.

In the R&D stage of the game, the managers of the firm choose the R&D effort for their respective firms. Again the solution is derived (for given values of \( \alpha_i \) and \( \alpha_j \)) by maximizing the performance measure \( U_i(\alpha_i, \alpha_j, x_i, x_j) \) of the manager, where the optimal choice of output (4) in the subsequent stage is already taken into account. From the first-order conditions we get the following R&D reaction functions \( RF_i(x_j) \):

\[
x_i = RF_i(x_j) = \max \left[ 0, \frac{2(2\alpha_i - \theta\alpha_j)(a + A(\alpha_j - 2\alpha_i))}{9\alpha_i br M} - 2(2\alpha_i - \theta\alpha_j)^2 + \frac{2(2\alpha_i - \theta\alpha_j)(2\theta\alpha_i - \alpha_j)}{9\alpha_i br M} - 2(2\alpha_i - \theta\alpha_j)^2 x_j \right].
\]

Note that for \( \alpha_i = \alpha_j \) the R&D efforts of the two firms are strategic complements for spillovers sufficiently high (\( \theta > 1/2 \)), and they are strategic substitutes for \( \theta < 1/2 \). Note also that \( \theta = 1/2 \) corresponds to a somehow degenerate case since the R&D expenditures in this case are constant and independent of the other firms choices (the same holds then for the production quantities). Solving for the optimal R&D expenses chosen by manager of firm \( i \) would yield the quantities \( x_i(\alpha_i, \alpha_j) \). \(^7\) Inserting this expression into (4) would yield the optimal quantity \( q_i(\alpha_i, \alpha_j) \) and inserting into the profit functions would give the profits of the owners solely as a function of the incentive parameters \( \alpha_i \) and \( \alpha_j \), i.e. \( \pi^P_i(\alpha_i, \alpha_j) \). \(^8\)

Observe that in our setup the owners/managers of both firms face exactly the same situation. Hence, we briefly look at the solution obtained in a symmetric equilibrium of the subgame. If we restrict ourselves to \( \alpha_i = \alpha_j = \alpha \) then we obtain from the general solutions above the managers’ choices in the symmetric equilibrium

\[
q_i = q_j = q = \frac{3r^M(a - A\alpha)}{9br M - 2\alpha(2\theta - 1)}
\]

\[
x_i = x_j = x = \frac{2(a - A\alpha)(2 - \theta)}{9br M - 2\alpha(2\theta - 1)}.
\]

These expressions coincide with the solutions provided by Zhang and Zhang (1995, 1997).

\(^7\)For the sake of completeness, we give the expressions for the optimal R&D expenses (neglecting the max-operator for the moment)

\[
x_i(\alpha_i, \alpha_j) = \frac{2(2\alpha_i - \theta\alpha_j)(\alpha_j (-4(a - A\alpha)\alpha_j + 3(a + A(-2\alpha_i + \alpha_j)))br M)}{(-6\alpha_i^2 br M^2 + 2\alpha_i br M(\theta - 6\alpha_i^2 + 4\alpha_j(\theta^2 - 1)) + 4\alpha_i\alpha_j(2\theta^2 - \alpha_j br M (\theta - 1) + 8\alpha_j^2 \theta (\theta^2 - 1)) - 4\alpha_i\alpha_j(-6\alpha_i^2 (\theta - 1) + \alpha_j (-4 + 3\theta^2 + \theta^4)))}
\]

\(^8\)Although these expressions can be calculated, they are rather complicated and we abstain from presenting them here.
3.2 The contracting stage: optimal choice of the incentive parameters $\alpha_i$ and $\alpha_j$

3.2.1 The results of Zhang and Zhang (1997)

In their paper Zhang and Zhang (1997) now solve the contracting stage of their game and characterize the optimal value of the incentive parameter $\alpha$ depending on the value of the spillover parameter $\theta$. Furthermore, they compare the R&D and production decisions and the profits in equilibrium of managerial and entrepreneurial firms.

In their proposition 1, Zhang and Zhang (1997, p. 394) assert that there exists a unique equilibrium value $\alpha^*$ for the incentive parameter. This value $\alpha^* < 1$ whenever spillovers are small ($\theta < \overline{\theta} = \max[0, (3 - \sqrt{1 + 6b\bar{r}M})/2])$, $\alpha^* > 1$ whenever spillovers are high ($\theta > \overline{\theta}$). In proposition 2 they state that managerial firms have higher R&D, higher output and lower prices than entrepreneurial firms whenever spillovers are small ($\theta < \overline{\theta}$), and vice versa for high spillovers. Zhang and Zhang (1997) provide the following reasoning for their results: There is a trade-off between the competition effect due to delegation and the spillover effect. For large spillovers the latter dominates the former and vice versa for small spillovers.

Although intuitive, the following counter-examples demonstrate that the results by Zhang and Zhang obviously do not give a characterization of the game where owners set the values of the incentive parameters noncooperatively.

- Counterexample for Proposition 1: Assume that owners select the incentive parameters noncooperatively and let $a = 100, b = 1, r^M = 10, \theta = 1 > \overline{\theta} = 0, A = 35.5$. Then by solving the first order conditions of the managers at stage 1 it can be easily verified that two optimal solutions for the contracting stage are obtained. The resulting R&D expenses, production quantities and efforts in equilibrium are:

<table>
<thead>
<tr>
<th>Solution 1</th>
<th>Solution 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^* = 0.215$</td>
<td>$\alpha^* = 0.44$</td>
</tr>
<tr>
<td>$q^M = 31.081$</td>
<td>$q^M = 28.692$</td>
</tr>
<tr>
<td>$x^M = 2.072$</td>
<td>$x^M = 1.913$</td>
</tr>
<tr>
<td>$G^M = 180$</td>
<td>$G^M = 295.65$</td>
</tr>
</tbody>
</table>

Accordingly, the solution is not unique and $\alpha^*$ is not larger than 1.

- Counterexample for Proposition 2: Assume that owners select the incentive parameters noncooperatively and let $a = 100, b = 1, r^M = 10, \theta = 1/2 > \overline{\theta} = 0, A = 35.5$. Then we again obtain two optimal solutions for the contracting stage. A comparison of the resulting R&D expenses, production quantities and efforts in equilibrium for the managerial firm and

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It can also be checked that the second order conditions for a maximum are fulfilled.
the entrepreneurial firm yields\textsuperscript{10}:

\begin{align*}
\text{Solution 1} & & \text{Solution 2} & & \alpha^O = 1 \\
\alpha^* = 0.066 & & \alpha^* = 0.559 & & \\
q^M = 32.659 & > & q^M = 27.485 & > & q^O = 22.632 \\
x^M = 3.266 & > & x^M = 2.749 & > & x^O = 2.263 \\
p^M = 34.683 & < & p^M = 45.03 & < & p^O = 54.737 \\
G^M = 79.97 & < & G^M = 337.464 & < & G^O = 486.58
\end{align*}

Hence, it is not true that managerial firms spend less on R&D and produce less in equilibrium.

**3.2.2 What went wrong?**

From these counterexamples it becomes obvious that Zhang and Zhang (1997) did not characterize the solution of the noncooperative R&D-game. Assuming that the owners choose their incentive parameters of the corresponding contracts noncooperatively, \( \alpha_i \) and \( \alpha_j \) have to be determined as the solutions of the system of first order conditions

\[
\frac{\partial \pi^D_i (\alpha_i, \alpha_j)}{\partial \alpha_i} \bigg|_{\alpha_i = \alpha_j = \alpha} = 0 \quad \frac{\partial \pi^D_j (\alpha_i, \alpha_j)}{\partial \alpha_j} \bigg|_{\alpha_i = \alpha_j = \alpha} = 0 \quad (7)
\]

when we restrict ourselves to symmetric equilibria. Looking at the paper of Zhang and Zhang (1997) and also Zhang and Zhang (1995), it becomes clear that, although they consider the noncooperative solutions for the production and the R&D stage, they set \( \alpha_i = \alpha_j = \alpha \) for the final stage (which results in \( \pi^D_i (\alpha) = \pi^D_j (\alpha) = \pi^D (\alpha) \)) and solve the resulting first order condition \( d\pi^D (\alpha)/d\alpha = 0 \). Economically speaking, this is equivalent to assuming that the two owners cooperate (i.e. maximize their joint profits) and select a value of the incentive parameter for a single contract which is then offered to both managers.\textsuperscript{11}

Since the assumption of cooperating owners is not along the lines of the early approaches by Fershtman and Judd (1987) and Vickers (1985), it seems to be more important to determine the noncooperative equilibrium. Unfortunately, it turns out that a closed-form solution cannot be given, not even under the assumption of symmetric equilibrium (7). The reason is that the first order condition for calculating the optimal values for the incentive parameter \( \alpha \) is equivalent to a fourth-degree polynomial in \( \alpha \). Moreover, it also seems impossible to characterize the roots of the polynomial in its general form due to

\textsuperscript{10}It can be checked again that the second order conditions for a maximum are fulfilled.

\textsuperscript{11}In their papers no explanation for this approach is given. Unfortunately, throughout their paper it also never becomes clear if this choice has been made on purpose or not. More bluntly, following their statements and results one gets the impression that the noncooperative equilibrium is considered. Only by looking at their working paper, Zhang and Zhang (1995), and re-calculating the derivations we finally were able to understand why our results were different from theirs.
the complexity of the coefficients. As a consequence, from an analytical point of view it seems impossible to provide a general characterization of the optimal solution of the game. Therefore, we suggest two possible ways to simplify the analysis and to provide a qualitative characterization of the equilibrium outcomes of this game. First, we assume discrete values for the spillover parameter $\theta$ and for the incentive parameter $\alpha$ and solve the resulting normal form games.\footnote{See for a similar approach Kräkel (2003). Note however, that Kräkel’s analysis focuses on a different type of market structure.} This enables us to gain insights into the qualitative features of the optimal strategies and has the additional advantage that we can also consider the possibility of asymmetric equilibria. As a second approach, we solve the first order conditions numerically for various sets of parameter values and try to gain further insights into the qualitative behavior of the outcomes.

3.3 The delegation stage

Zhang and Zhang (1997) did not explicitly consider the owners’ options whether to delegate or not. However, this obviously becomes an important issue given that they state in their third proposition that, “Managerial firms have lower equilibrium profits than their entrepreneurial counterparts, irrespective of the level of spillovers.” (p. 394). If this result would hold true, then delegation would never be optimal, somehow questioning the whole analysis. The following counterexample shows that also their proposition 3 is incorrect, given that we assume that owners play noncooperatively at the contracting stage.

- Counterexample for Proposition 3: Assume that owners select the incentive parameters noncooperatively and let $a = 100, b = 1, r^M = 10, \theta = 3/4 > \overline{\theta} = 0, A = 5.1$. Then we obtain a unique optimal solution for the contracting stage. A comparison of the resulting R&D expenses, production quantities, and efforts in equilibrium for the managerial firm and the owner-managed firm yields:\footnote{Again, the second order conditions for a maximum are fulfilled.}

\[
\begin{align*}
\alpha^* &= 15.282 & \alpha^* &= 1 \\
q^M &= 28.6 & < & q^O &= 33.25 \\
x^M &= 2.38 & < & x^O &= 2.77 \\
p^M &= 42.8 & > & p^O &= 33.5 \\
G^M &= 1169.1 & > & G^O &= 1067.2
\end{align*}
\]

In this symmetric equilibrium, the owners select a high positive weight on profits and a high negative weight on sales in the contract of their respective manager. This choice results in punishing the manager for aggressive behavior on the market and keeps prices higher, which in turn leads to higher profits for the owners.
4 The discrete case

As pointed out above, a rigorous analytical characterization of the equilibrium outcomes, in particular in the contracting stage, seems impossible. In this section we investigate the discrete case, i.e. we assume discrete values for the incentive parameter and the level of spillovers and characterize the equilibria of the resulting game. In this section we assume that $\alpha_i, \alpha_j \in \{1/2, 1, 3/2\}$\textsuperscript{14}. This specification allows us to analyze three different settings. First, setting $\alpha_i = 1$ reflects profit maximization and coincides with the owner’s solution. Here manager $i$’s performance based payment only includes the profit. Secondly, $\alpha_i = 1/2$ directs the manager away from pure profit maximization and awards the manager for higher sales revenues. Thirdly, $\alpha_i = 3/2$ puts more weight on the profit as compared to the situation of pure profit maximization and punishes for sales. Moreover, in this section the spillover parameter $\theta$ is restricted to values from the set $\{0, 1/4, 1/2, 3/4, 1\}$. By this it is possible to look at the extreme cases of no spillovers ($\theta = 0$) and maximum spillovers ($\theta = 1$), and also to consider the case of “medium”, “small”, and “large” spillovers. Analyzing the resulting situations for different values of the incentive parameters and spillovers, we can identify which contract works best as a strategic delegation device for the owners.

Solving the subgame described above for $\alpha_i, \alpha_j \in \{1/2, 1, 3/2\}$ and $\theta \in \{0, 1/4, 1/2, 3/4, 1\}$ leads to the following results.

- **No spillovers ($\theta = 0$)**
  The optimal choices of R&D expenses and production quantity by the manager of firm $i$ are given by
  \[
  x_i(\alpha_i, \alpha_j) = \max\left[0, \frac{-16\alpha_j(a - \alpha_i) + 12br^M(a + A(\alpha_j - 2\alpha_i))}{16a_0^2x_j - 24(a_0 + \alpha_j)br^M + 27b^2r^M^2}\right],
  \]
  \[
  q_i(\alpha_i, \alpha_j) = \frac{3p^M}{4} - x_i(\alpha_i, \alpha_j).
  \]
  The profits of the owners for the nine combinations of the incentive parameters $\alpha_i, \alpha_j$ are given in Table 1. We only give the values above the diagonal since the profit of owners $i$ and $j$ are just interchanged for value combinations of the incentive parameters below the diagonal.

- **Small spillovers ($\theta = 1/4$)**
  The optimal choices of R&D expenses and production quantity by the manager of firm $i$ are given by
  \[
  x_i(\alpha_i, \alpha_j) = \max\left[0, \frac{4(8\alpha_j - \alpha_i)(\alpha_i - 8\alpha_j)(a(a_0 - 4\alpha_j) + 3a_0(a_i - 4\alpha_j) - 24a_j(a + A(\alpha_j - 2\alpha_i))br^M)}{8\alpha_0^2\alpha_j^2\alpha_i^2 - 8\alpha_i^3 - 8\alpha_j^3 + 8(a_i + \alpha_j)(\alpha_i^2 + 47\alpha_i\alpha_j + \alpha_j^2)br^M - 576\alpha_0\alpha_ibr^M} \right],
  \]
  \[
  q_i(\alpha_i, \alpha_j) = \frac{6\alpha_i p^M}{8\alpha_i - \alpha_j}x_i(\alpha_i, \alpha_j).
  \]

\textsuperscript{14}See for a similar assumption Kräkel (2003), p. 6f.
The profits of the owners for the nine combinations of the incentive parameters $\alpha_i, \alpha_j$ are given in Table 2.

- **Medium spillovers ($\theta = 1/2$)**
  The optimal choices of R&D expenses and production quantity by the manager of firm $i$ are given by

  $$x_i(\alpha_i, \alpha_j) = \max \left\{ 0, \frac{2(4\alpha_i-\alpha_j)((\alpha_i-4\alpha_j)(a(\alpha_i-2\alpha_j)+\alpha_i\alpha_j+6\alpha_j(a+A(\alpha_j-2\alpha_i)))br^M}{3\alpha_i\alpha_j(4\alpha_i-\alpha_j)+6(\alpha_i+\alpha_j)(\alpha_i^2+7\alpha_i\alpha_j+\alpha_j^2)br^M-108\alpha_i\alpha_jb^M} \right\}$$

  $$q_i(\alpha_i, \alpha_j) = \frac{3\alpha_i r^M}{4\alpha_i - \alpha_j} x_i(\alpha_i, \alpha_j)$$

  The profits of the owners for the nine combinations of the incentive parameters $\alpha_i, \alpha_j$ are given in Table 3.

- **Large spillovers ($\theta = 3/4$)**
  The optimal choices of R&D expenses and production quantity by the manager of firm $i$ are given by

  $$x_i(\alpha_i, \alpha_j) = \max \left\{ 0, \frac{4(3\alpha_i-3\alpha_j)(a(3\alpha_i-8\alpha_j)+\alpha_i\alpha_j+24\alpha_j(a+A(\alpha_j-2\alpha_i)))br^M}{7\alpha_i\alpha_j(3\alpha_i-8\alpha_j)+24(\alpha_i+\alpha_j)(9\alpha_i^2+7\alpha_i\alpha_j+9\alpha_j^2)br^M-1728\alpha_i\alpha_jb^M} \right\}$$

  $$q_i(\alpha_i, \alpha_j) = \frac{6\alpha_i r^M}{8\alpha_i - 3\alpha_j} x_i(\alpha_i, \alpha_j)$$

  The profits of the owners for the nine combinations of the incentive parameters $\alpha_i, \alpha_j$ are given in Table 4.

- **Maximal spillovers ($\theta = 1$)**
  The optimal choices of R&D expenses and production quantity by the manager of firm $i$ are given by

  $$x_i(\alpha_i, \alpha_j) = \max \left\{ 0, \frac{2(2\alpha_i-\alpha_j)(2a(\alpha_i-2\alpha_j)(\alpha_i-\alpha_j)-3\alpha_j(a+A(\alpha_j-2\alpha_i)))br^M}{3br^M(2(\alpha_i^2+\alpha_j^2)-9\alpha_i\alpha_jbr^M)} \right\}$$

  $$q_i(\alpha_i, \alpha_j) = \frac{3\alpha_i r^M}{4\alpha_i - 2\alpha_j} x_i(\alpha_i, \alpha_j)$$

  The profits of the owners for the nine combinations of the incentive parameters $\alpha_i, \alpha_j$ are given in Table 5.

Clearly, in order to guarantee that prices, quantities, R&D expenses, effective production unit costs and profits of the owners are nonnegative, we have to limit the possible set of values of the parameters $a, A, b$ and $r^M$. In order to demonstrate the economic meaning of such restrictions, we first study the symmetric case (see similarly, Zhang and Zhang 1997). From (6) we can see
that the following conditions guarantee that quantities and R&D efforts are positive

\[ a > A \alpha \]

\[ 9br_{2}M - 2\alpha(2 - \theta)(1 + \theta) > 0. \]

The effective production costs \( A - x_i - \theta x_j \) have to be positive too. From \( A > x_i + \theta x_j \) we get using (6) the condition

\[ 9br_{2}M A - 2\alpha(2 - \theta)(1 + \theta) > 0. \]

Finally, the stability of the equilibrium in the R&D stage is only guaranteed if the R&D reaction functions cross each other correctly. This is ensured by the following two conditions

\[ 3br_{2}M - 2\alpha(2 - \theta)(1 - \theta) > 0 \]

\[ 9br_{2}M - 2\alpha(2 - \theta)^2 > 0. \]

Using the values of \( \alpha_1, \alpha_2 \in \{1/2, 1, 3/2\} \) and \( \theta \in \{0, 1/4, 1/2, 3/4, 1\} \) and checking for redundant restrictions, it turns out that if the conditions

\[ \frac{2}{3} a > A > \frac{1}{2br_{2}M} a \]

are satisfied, then quantities and (effective) production costs are positive. The economic interpretation is straightforward and characterizes an economic trade-off in this model. On the one hand, the basic production costs \( A \) have to be small enough (in comparison to the highest value or price \( a \)) such that production is economically viable. On the other hand, the effective production costs have to be positive too. This can be guaranteed only if the basic production costs \( A \) are sufficiently large.

It is easy to see, that the conditions in (11) do not ensure the positivity of profits, however (e.g. for \( \theta = 0 \) the profits of the owners for \( \alpha_i = \alpha_j = 1/2 \) are positive if and only if \( A < a/2 \); see Table 1). Of course, they also do not ensure positivity of quantities, expenses and profits in the other, non-symmetric cases. Therefore, in the following proposition we give refined conditions such that all decision variables, costs and profits are positive.

Proposition 1: Let \( br_{2}M > 2 \). Then for \( \alpha_1, \alpha_2 \in \{1/2, 1, 3/2\} \) and \( \theta \in \{0, 1/4, 1/2, 3/4, 1\} \), production quantities and prices, R&D expenditures, effective production costs, and profits of the owners are positive if

\[ A_L < A < A_U, \]

15The stability condition requires that the product of the slopes of the two R&D reaction functions is less than one. In other words, a sufficient condition is \(|dr_1/dx_2|dr_2/dx_1| < 1\), see e.g. Fudenberg and Tirole (1993).

16This becomes even more obvious if we take the profits along the diagonal in Tables 1-5 into account.

12
where $A_L = \frac{110br^M - 59}{120br^M - 25} \alpha$ and $A_U = \frac{6br^M - 4}{130br^M - 6} \alpha$.

**Proof:** The proof of this proposition is based on straightforward – although tedious – calculations. One has to derive the set of conditions such that all expressions for quantities, prices, effective production costs, etc. are positive and then check which inequality gives the tightest bounds. The only two cases which deserve particular attention are (i) $\theta = 3/4, \alpha_1 = 1/2, \alpha_2 = 3/2$ and (ii) $\theta = 1, \alpha_1 = 1/2, \alpha_2 = 3/2$. In case (i) the R&D reaction functions given in (5) assume the form

$$x_1 = \max \left[ 0, - \frac{8a + 4A}{144br^M - 1} + \frac{6}{144br^M - 1} x_2 \right]$$
$$x_2 = \max \left[ 0, \frac{28(2a - 5A)}{3(48br^M - 49)} + \frac{7}{3(48br^M - 49)} x_1 \right]$$

and the resulting intersection point is

$$x_1 = 0; \quad x_2 = \frac{28(2a - 5A)}{3(48br^M - 49)}.$$ 

The profits, quantities and effective production costs have to be calculated using these expressions for the R&D expenditures in this particular case. Similarly, in case (ii) the R&D reaction functions given in (5) assume the form

$$x_1 = \max \left[ 0, - \frac{2a + A}{9br^M - 1} + \frac{1}{9br^M - 1} x_2 \right]$$
$$x_2 = \max \left[ 0, \frac{5(2a - 5A)}{27br^M - 25} + \frac{25}{27br^M - 25} x_1 \right]$$

and the resulting intersection point is

$$x_1 = 0; \quad x_2 = \frac{5(2a - 5A)}{27br^M - 25}.$$ 

Again, the profits, quantities and effective production costs have to be calculated using these expressions for the R&D expenditures.

Now, since we know for which values of the model parameters the decision variables and payoffs are positive, it is possible to analyze which selection of the incentive parameter is optimal for the owners in terms of their profits. Each owner can pick $\alpha_i$ from the set $\{1/2, 1, 3/2\}$ and depending on the spillover parameter $\theta \in \{0, 1/4, 1/2, 3/4, 1\}$ the combination of the strategy choices of the owners leads to the profits given in Tables 1-5. In other words, these tables represent the payoffs of the R&D game in normal form and we have to determine which strategy combination $(\alpha_1, \alpha_2)$ is a Nash-equilibrium (together with the corresponding expressions for production quantities and R&D efforts this then
yields the subgame perfect equilibrium outcome). If there are no spillovers, i.e. \( \theta = 0 \), then we can give the following general result for the normal form game.

**Proposition 2:** Consider the case where no spillovers exist, \( \theta = 0 \). Let \( b^M_r > 2 \) and let condition (12) hold. Then, the choice of \( \alpha_i = 1/2 \) (\( \alpha_j = 1/2 \)) is a dominant strategy for player \( i \) (player \( j \)). Hence, \( (\alpha_1 = 1/2, \alpha_2 = 1/2) \) is the unique equilibrium of the normal form game.

**Proof:** Again, the proof is straightforward. All that has to be shown is that no matter what player \( i \) does, player \( j \) selects \( \alpha_j = 1/2 \). This can be proven by checking that the profit differences between equilibrium play and deviation are negative.

It is well-known that in a game without R&D and spillovers the owner of a firm can gain a Stackelberg-leader position if production is delegated to a manager (Fershtman and Judd 1987). The manager serves as a self-committing device for the owner. More aggressive behavior is induced by choosing the incentive parameter of the manager’s contract \( \alpha_i < 1 \). However, if both firms delegate, then they find themselves in some sort of prisoner’s dilemma situation. Both firms induce their managers to act aggressively on the market by choosing \( \alpha_i < 1 \) and their profits in equilibrium are smaller than the profits without delegation \( (\alpha_1 = \alpha_2 = 1) \). Obviously, the same forces are at play here. It is easy to see that the owners would be better off if they would not delegate R&D and production given that R&D costs are the same for owners and managers, i.e. \( \pi^O = \pi^M \).

If spillovers exist, then – from an incentive point of view – the outcome might be different. The reason is that the existence of spillovers reduces the incentive of doing R&D for the players, since every player wants to free ride on the R&D expenditures of the competitor. However, lower R&D expenditures keep production costs high and this results in less aggressive behavior on the market. We give two propositions which illustrate the type of results which can be obtained for the discrete model.

**Proposition 3:** Consider the case of medium spillovers, \( \theta = 1/2 \). Let \( b^M_r > 2 \) and let condition (12) hold. Then for \( A = A_L \), the following holds.

(i) The resulting normal form game has 3 equilibria, namely \( (\alpha_1 = 1/2, \alpha_2 = 1/2), (\alpha_1 = 1, \alpha_2 = 1), \) and \( (\alpha_1 = 3/2, \alpha_2 = 3/2) \).

(ii) In the equilibrium \( (\alpha_1 = 3/2, \alpha_2 = 3/2) \) the profits for the owners of the managerial firm are higher than the profits of the entrepreneurial firm \( (\alpha_1 = 1, \alpha_2 = 1) \).

**Proposition 4:** Consider the case of small spillovers, \( \theta = 1/4 \). Let \( b^M_r > 2 \) and let condition (12) hold. Then for \( A = A_L \), the following holds.

\[ \text{Just compare the profits } \pi_i^{ND} \text{ for } \theta = 0 \text{ with the entries in Table 1 for } \alpha_1 = \alpha_2 = 1/2. \]

We will discuss the case where the R&D costs differ in more detail later on.
(i) The resulting normal form game has 2 equilibria, namely \((\alpha_1 = 1/2, \alpha_2 = 1/2)\) and \((\alpha_1 = 3/2, \alpha_2 = 3/2)\).

(ii) In the equilibrium \((\alpha_1 = 3/2, \alpha_2 = 3/2)\) the profits for the owners of the managerial firm are higher than the profits of the entrepreneurial firm \((\alpha_1 = 1, \alpha_2 = 1)\).

Proof of Propositions 3 and 4: In order to show the claims of the propositions, the first step is to evaluate the expressions for the profits, see Tables 2 and 3 for \(A = A_L\). The second step is then to show that the resulting expressions in the parameters \(a, b,\) and \(r^M\) are positive given the parameter restrictions of the propositions. Moreover, it has to be proven that the difference in profits for equilibrium choices (along the diagonals) and deviations are positive. This then proves the claims.

From our analysis we can now draw the following preliminary conclusions. First, every equilibrium of the R&D delegation game with spillovers is symmetric. Both owners choose the same level of incentives for their managers. In our model situations as in Kräkel (2003), where the equilibrium is asymmetric, do not occur. Second, the symmetric equilibrium need not be unique. Third, delegation might be even beneficial for the owners, even if R&D costs are the same for owners and managers. Since the profits for both owners are the highest for \((\alpha_1 = 3/2, \alpha_2 = 3/2)\) in the cases described by the previous Propositions, coordination on the high-profit equilibrium is plausible since this equilibrium serves as a focal point in these games.

5 The continuous case: a numerical analysis

In the previous section we could gain some insights into the qualitative properties of the equilibrium outcomes by characterizing the equilibria for discrete values of the incentive parameters and the level of spillovers. We now use this increased understanding of the model to choose values for the parameters according to Proposition 1 and to calculate the equilibrium choices numerically for all values of \(\theta \in [0, 1]\). That is, for given values of the parameters \(a, b, A,\) and \(r^M\), we solve the first order condition (7) for the optimal values \(\alpha^*\) of the incentive parameter\(^{18}\) \(\alpha\) for a particular value of \(\theta \in [0, 1]\) (starting from \(\theta = 0\)). We check (i) which of the solutions for \(\alpha\) is real-valued, (ii) if the second order condition for a profit maximum is fulfilled, and (iii) if profit, R&D expenses, effective production costs, prices and quantities are positive. If these questions are answered affirmatively, we keep this value \(\alpha^*(\theta)\), together with \(G(\alpha^*(\theta)), x(\alpha^*(\theta))\), and \(q(\alpha^*(\theta))\). We repeat this procedure for \(n\) values of \(\theta \in [0, 1]\).

In Figure 1 we show the values of the incentive parameter \(\alpha^*\), the profits, the R&D expenses, and the quantities in equilibrium for \(\theta \in [0, 1]\). The basic

\(^{18}\)Recall that the optimal values \(\alpha^*\) can be calculated as the solutions of a polynomial of degree 4. Hence, in general, we get 4 solutions, which can be real-values or (conjugate) complex.
production costs are \( A = 35.5 \), the market parameters are \( a = 100, b = 1 \) and the R&D cost parameter of the manager is \( r^M = 10 \). Note that this coincides with the set of parameters we used for the counterexamples above and that this set fulfills the conditions of Proposition 1. For the sake of comparison we also show the solutions for the non-delegation case, see the expressions in (3). Figure 1 reveals that the equilibrium value \( \alpha^* \) of the incentive parameter \( \alpha \) may not be unique. For values of \( \theta \) greater than about 0.35 we obtain two real-valued solutions. Note that one solution is decreasing in \( \theta \) (upper branch), whereas the other solution is increasing in \( \theta \). Hence, the solution does not seem to possess any uniqueness or monotonicity properties. Panels 2-4 of Figure 1 illustrate that profits for the non-delegation case are higher than in the delegation equilibria (which makes non-delegation the preferred choice assuming \( r^M = r^O \)), that managerial firms spend more on R&D and produce more than entrepreneurial firms independent of the level of spillovers.

We now select a value of the basic production costs close to \( A_L \), namely \( A = 5.1 \). All other values remain the same. In Figure 2 we show the resulting equilibrium values \( \alpha^* \). Here the following qualitative feature of the solutions can be seen. For low levels of spillovers \( (\theta < 1/2) \) the equilibrium value \( \alpha^* \) is increasing in \( \theta \), whereas it is decreasing for high levels of spillovers \( (\theta > 1/2) \). Note that the manager is punished for sales. Profits in equilibrium are higher than in the non-delegation case, R&D expenditures and quantities are lower.

We finally demonstrate that mixed situations exist, where the profits for managerial firms are higher than for entrepreneurial firms, and the former also spend more on R&D and produce higher quantities. We consider \( a = 1, b = 0.1, r^M = 1, A = 0.76 \). Note that for these values of the parameters the conditions of Proposition 1 are not fulfilled. Figure 3 depicts the optimal values \( \alpha^* \) for \( \theta \in [0, 1] \). Throughout the whole range of spillovers, the manager is punished for sales. The profits, R&D expenditures and the produced quantities in equilibrium for a managerial firm are higher than the corresponding values of the entrepreneurial firm (note that the values are negative; therefore, an entrepreneurial firm would not invest and not produce, exiting the market and consequently the profit would be zero).

6 Further results and extensions of the model

In this section we are turning to some possible extensions of the model.

- **Asymmetric R&D costs:** Proposition 2 shows that delegation is never beneficial for the owners when no spillovers exist. In situations where spillovers do exist, our numerical investigations showed that the same result might hold. However, if all owners are worse off by delegating R&D and production, delegation will not occur. However, delegation might be beneficial if the manager has some characteristics which differ from those of the owner of the firm. Such a difference in characteristics can be incorporated in the costs for R&D. In our setup the R&D costs of the manager and owner
would then be considered as different, i.e. \( r^M < r^O \). Consider now the case of maximal spillover \((\theta = 1)\) and let \( a = 100, b = 1, r^M = 10 \) and \( A = 10 \). In this situation in the normal form game derived from Table 5 the unique equilibrium is \((\alpha_1 = 1/2, \alpha_2 = 1/2)\) and the profits for the owners in equilibrium is \( \pi^D_1 = \pi^O_2 \approx 933.56 \). Calculating the profits if the owners carry out R&D and production themselves (see the last line in (3)) for the same values of the parameters, it turns out that \( \pi^{ND}_1 = \pi^{ND}_2 < 933.56 \) for \( r^O > 18.47 \). Hence, if R&D costs of the owners are about twice as high, then delegation is beneficial.

- **Asymmetric spillovers:** Another difference between manager and owner may arise in the size of the spillover parameter \( \theta \). A manager who is in charge of the R&D task certainly has accumulated an expertise which enables him to make better use of the R&D knowledge which spills over from the rival firm. This could be imagined as switching from a profit table with lower \( \theta \) to a profit table with higher \( \theta \). The point that the manager is more able to absorb and use knowledge spilled over from the rival firm is somehow reminiscent of the notion of absorption capacity (see Cohen and Levinthal 1989). In order for firms being able to use knowledge from other firms, they have to expend resources to build up capacity which then allows to absorb and use this knowledge to their advantage (see Kaiser (2002) or Grünfeld (2003)).

- **R&D set up costs:** In our model the marginal costs of R&D are linear. This certainly is a simplification, in particular when one thinks of the set-up costs involved when firms start R&D activities. A different type of R&D cost function, e.g. \( x + x^2 \), takes such an effect into account. The positive marginal costs for R&D at \( x = 0 \) provide some kind of entry barrier.

- **Different Performance measures:** In our model we assume that incentive contracts are based on sales (or equivalently on sales revenue) and profits. In doing that we followed most of the literature on strategic delegation. However, other authors study the effect of other (additional) performance measures, e.g. relative profits, market shares, etc. For work along these lines see Aggarwal and Samwick (1999) or Zitzewitz (2001).

- **Job design:** In this paper (and the literature on strategic delegation in an R&D context) it is assumed that either the owner delegates both tasks (production and R&D) or carries out both tasks himself. However, the two tasks are quite different in character. R&D have a strategic orientation, whereas production occurs on an operational level. Therefore, it makes sense to consider the mixed case where either the owner carries out R&D and only delegates the production task, or hires a different manager for each task (an innovation expert and a production line manager) and offers different compensation contracts to the two employees. In the literature on job design (see e.g. Holmstrom and Milgrom (1991) or Hemmer (1995))
3.1 Market structure
It might be interesting to consider different market structures and their influence on the outcomes. From the industrial organization literature we know that the characteristics of the decision variables have a strong influence on the result. For example, in a homogeneous-products industry with oligopolistic competition, firms’ profits are eroded when price competition is considered, whereas they are positive in a quantity setting framework. It might be interesting to consider the effects of delegation in an R&D model where the manager is responsible for setting prices instead of production quantities (see Miller and Pazgal (2001) for an interesting equivalence result for price and quantity competition).

7 Conclusions
The question of strategic delegation has attracted much attention since the seminal papers of Vickers (1985) and Fershtman and Judd (1987). Recently several papers focus on the effects of strategic delegation in the context of R&D decisions by building on Zhang and Zhang (1997), who were the first to combine these two streams of research. Looking at the findings of Zhang and Zhang (1997) a major puzzle remains which has served as a starting point of the analysis of the present paper, namely that the owners of a firm are always better off by not delegating production and R&D decisions to managers. So why should there be a need for managers in this setting and why should delegation happen at all? For this reason we have reconsidered the analysis of Zhang and Zhang (1997) to gain insight into this question. Re-analyzing their paper has shown that they derive their results under the (implicit) assumption that at the first stage of the game the owners of the firms cooperate in designing the managerial compensation contracts. This implicit assumption is not in line with the literature on strategic delegation, which emphasizes the fact that delegation works as a self-committing device when the owners of the firms act noncooperatively. Moreover, this assumption seems strange, as all other stages in the game are played noncooperatively. For this reason we have studied the R&D delegation model under the assumption that owners determine the incentive parameters of the contracts noncooperatively at the contracting stage. Additionally, we have introduced an important stage at the beginning of the game, considering the owner’s option whether to delegate or not. It turns out that due to the complexity of the problem a closed-form solution can not be given for the optimal values of the incentive parameter $\alpha$. We suggest two possibilities to deal with this complexity. The first is to study the model for discrete values of the exogenous parameters, the second is to employ numerical methods. By using such techniques we have shown that the general findings derived by Zhang and Zhang (1997) do not hold in the noncooperative setting.

19See, for example, Zhang (2003), Kräkel (2003) or Lambertini (2004).
From our analysis of the noncooperative setting we can derive the following conclusions. The equilibrium of the R&D delegation game is symmetric, but the equilibrium need not be unique. Additionally it can be shown that there exist situations where delegation is beneficial for the owners of the firm. The profits which can be achieved in the delegation setting exceed the profits in the case where the owners decide themselves on production quantities and R&D expenditures. This finding can be observed even in the setting where firm owners and managers have the same characteristics with respect to costs and benefits of carrying out R&D. In addition to these results, we have also discussed how different characteristics of owners and managers, like different costs of carrying out R&D or different abilities to absorb R&D spillovers, affect the benefits of delegation. Since we find the combination of strategic delegation and R&D a fruitful field for future research, we have discussed several open topics. We hope to address some of these issues in future papers.
References


Holmstrom, B; Milgrom, P.: Multitask principal-agent analysis: Incentive contracts, asset ownership, and job design, Journal of Law, Economics, and Organization, 7, 524 - 552


Table 1: Owner’s profits for $\theta = 0$

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2 = \frac{1}{2}$</th>
<th>$\alpha_2 = 1$</th>
<th>$\alpha_2 = \frac{3}{2}$</th>
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<td>$\frac{1}{2}$</td>
<td>$\frac{(a-2\alpha)(2a-\alpha)^M}{18br^M-4}$, $\frac{(a-2\alpha)(2a-\alpha)^M}{18br^M-4}$</td>
<td>$\frac{r^M(2a+4(3br^M-4))(2a(3br^M-4)9br^M-2)+A(9br^M(16-9br^M)-32))}{2(8+9br^M(3br^M-4))^2}$, $\frac{r^M(9br^M-8)/4(4-9br^M)+A(6br^M-4))}{2(8+9br^M(3br^M-4))^2}$</td>
<td>$\frac{r^M(-4a+2+4(2a+4)(br^M)(9br^M-2)+A(br^M(32-9br^M-8)))}{2(4+br^M(9br^M-2))^2}$, $\frac{r^M(3.4(2-5br^M)+A(6br^M-4))}/(3br^M-2)+A(6br^M-4))}{2(4+br^M(9br^M-2))^2}$</td>
</tr>
<tr>
<td>$1$</td>
<td>$\frac{(a-\alpha)^2r^M(9br^M-8)}{(9br^M-4)^2}$, $\frac{(a-\alpha)^2r^M(9br^M-8)}{(9br^M-4)^2}$</td>
<td>$\frac{r^M(9br^M-8)(4a-4+(-2a+4)br^M)}{4(8+br^M(9br^M-20))^2}$, $\frac{r^M(-4a+6+3(2-4)br^M)(2a(3br^M-4)(9br^M-14)+A(32+br^M(9br^M-32))))}{18(8+br^M(9br^M-20))^2}$</td>
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<tr>
<td>$\frac{3}{2}$</td>
<td></td>
<td></td>
<td>$\frac{(2a-3\alpha)r^M(2a+3br^M-14))}{18(3br^M-2)^2}$, $\frac{(2a-3\alpha)r^M(2a+3br^M-14))}{18(3br^M-2)^2}$</td>
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<td>( \alpha_2 )</td>
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<td>( \frac{1}{2} )</td>
<td>( \frac{8(2a-A)M}{(35-144hrM)^2} )</td>
<td>( \frac{4rM(15.4+a(32brM-35))(a(3+8hrM(15-16hrM))+6.4(5+2brM(-19+16hrM)))}{9(32hrM-25)^2(4brM-1)^2} )</td>
<td>( \frac{8rM(-506a+207.4+144(2a+4)hrM)(a(-2875+576hrM(3(16hrM-55)-54))}\cdot9.4(375+16hrM(144hrM-353)))}{9(1725+64hrM(108hrM-151))^2} )</td>
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<tr>
<td>1</td>
<td>( \frac{8(a-A)^2rM(72hrM-49)}{(72hrM-35)^2} )</td>
<td>( \frac{8(a-A)^2rM(72hrM-49)}{(72hrM-35)^2} )</td>
<td>( \frac{8rM(288hrM-169)(110a-99.4+36(-2a+4)hrM)}{9(2145+4brM(864hrM-1475))^2} )</td>
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<tr>
<td>( \frac{3}{2} )</td>
<td>( \frac{8(2a-3.4)rM(147.4+a(144hrM-203))}{9(48hrM-35)^2} )</td>
<td>( \frac{8(2a-3.4)rM(147.4+a(144hrM-203))}{9(48hrM-35)^2} )</td>
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Table 3: Owner’s profits for \( \theta = \frac{1}{2} \)

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<thead>
<tr>
<th>( \alpha_1 )</th>
<th>( \alpha_2 = \frac{1}{2} )</th>
<th>( \alpha_2 = 1 )</th>
<th>( \alpha_2 = \frac{3}{2} )</th>
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<tr>
<td></td>
<td>( \frac{2(2a-A)r^H(2a+A+4abr^M-8abr^M)}{9(4br^M-1)^2} ), ( \frac{9(7+3hr^M)(24br^M-19)}{2r^H(72br^M-49)(4+6ahr^M-94br^M)} )</td>
<td>( \frac{2r^M(-110a+33A+36(2a+A)r^H)(-33A+36abr^M(53-36hr^M)+a(-187+144hr^M(9hr^M-10)))}{9(33+16hr^M(27hr^M-31))^2} ), ( \frac{2r^H(4(3-60hr^M)+a(24br^M+2)(a(55+48hr^M(81hr^M-110))-3a(121+4hr^M(324hr^M-569)))}{9(33+16hr^M(27hr^M-31))^2} )</td>
<td>( \frac{2(2a-3r)hr^M(3+4a(4hr^M-5))}{9(4br^M-3)^2} ), ( \frac{2(2a-3r)hr^M(3+4a(4hr^M-5))}{9(4br^M-3)^2} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{2(a-A)r^Hr^M}{18hr^M-9} ), ( \frac{2(a-A)r^H}{18hr^M-9} )</td>
<td>( \frac{2r^M(72hr^M-25)(2a+154+9(-2a+A)r^H)^2}{9(75+hr^M(216hr^M-275))^2} ), ( \frac{r^M(-5a+15+24(a-A)r^H)(-3a(250+6hr^M(324hr^M-755)))+a(925+6hr^M(324hr^M-515)))}{9(75+hr^M(216hr^M-275))^2} )</td>
<td>( \frac{2(2a-3-4)r^H(3+4a(4hr^M-5))}{9(4br^M-3)^2} ), ( \frac{2(2a-3-4)r^H(3+4a(4hr^M-5))}{9(4br^M-3)^2} )</td>
</tr>
</tbody>
</table>
Table 4: Owner’s profits for $\theta = \frac{3}{4}$ (Note: the profits for $\alpha_1 = \frac{1}{2}, \alpha_2 = \frac{1}{2}$) have been obtained by setting $x_1 = 0, x_2 = \frac{28(2a-54)}{3(48br_{M-49})}$

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2 = \frac{1}{2}$</th>
<th>$\alpha_2 = 1$</th>
<th>$\alpha_2 = \frac{3}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = \frac{1}{2}$</td>
<td>$\frac{8(2a-34)r^M(55a+284+4br_{M-24})}{(35-144br_{M})^2}$, [4a^M(13a+a(96br_{M-65}))] (26a+36br_{M}(43+144br_{M})) (a(143+24br_{M}(23-144br_{M}))) (\frac{(91+36br_{M}(96br_{M-59}))^2}{(19+36br_{M}(96br_{M-59}))^2})</td>
<td>$\frac{8(2a-34)r^M(55a+284+4br_{M-24})}{(35-144br_{M})^2}$, [4a^M(13a+a(96br_{M-65}))] (26a+36br_{M}(43+144br_{M})) (a(143+24br_{M}(23-144br_{M}))) (\frac{(91+36br_{M}(96br_{M-59}))^2}{(19+36br_{M}(96br_{M-59}))^2})</td>
<td>$\frac{8(2a-34)r^M(72br_{M-25})}{(72br_{M-35})^2}$, [4a^M(7(a+34)+96(a-24)+24br_{M}(91-48br_{M})+a(35+88br_{M}(48br_{M-55}))) (288br_{M-49}) (\frac{(288br_{M-49})^2}{(4br_{M-3})^2})</td>
</tr>
<tr>
<td>$\alpha_1 = 1$</td>
<td>$\frac{8(2a-34)r^M(72br_{M-25})}{(72br_{M-35})^2}$, [4a^M(7(a+34)+96(a-24)+24br_{M}(91-48br_{M})+a(35+88br_{M}(48br_{M-55}))) (288br_{M-49}) (\frac{(288br_{M-49})^2}{(4br_{M-3})^2})</td>
<td>$\frac{8(2a-34)r^M(72br_{M-25})}{(72br_{M-35})^2}$, [4a^M(7(a+34)+96(a-24)+24br_{M}(91-48br_{M})+a(35+88br_{M}(48br_{M-55}))) (288br_{M-49}) (\frac{(288br_{M-49})^2}{(4br_{M-3})^2})</td>
<td>$\frac{8(2a-34)r^M(75a+a(144br_{M-155}))}{(48br_{M-35})^2}$, [4a^M(7(a+34)+96(a-24)+24br_{M}(91-48br_{M})+a(35+88br_{M}(48br_{M-55}))) (288br_{M-49}) (\frac{(288br_{M-49})^2}{(4br_{M-3})^2})</td>
</tr>
<tr>
<td>$\alpha_1 = \frac{3}{2}$</td>
<td>$\frac{8(2a-34)r^M(72br_{M-25})}{(72br_{M-35})^2}$, [4a^M(7(a+34)+96(a-24)+24br_{M}(91-48br_{M})+a(35+88br_{M}(48br_{M-55}))) (288br_{M-49}) (\frac{(288br_{M-49})^2}{(4br_{M-3})^2})</td>
<td>$\frac{8(2a-34)r^M(72br_{M-25})}{(72br_{M-35})^2}$, [4a^M(7(a+34)+96(a-24)+24br_{M}(91-48br_{M})+a(35+88br_{M}(48br_{M-55}))) (288br_{M-49}) (\frac{(288br_{M-49})^2}{(4br_{M-3})^2})</td>
<td>$\frac{8(2a-34)r^M(75a+a(144br_{M-155}))}{(48br_{M-35})^2}$, [4a^M(7(a+34)+96(a-24)+24br_{M}(91-48br_{M})+a(35+88br_{M}(48br_{M-55}))) (288br_{M-49}) (\frac{(288br_{M-49})^2}{(4br_{M-3})^2})</td>
</tr>
</tbody>
</table>
Table 5: Owner’s profits for $\theta = 1$ (Note: the profits for $(\alpha_1 = \frac{1}{2}, \alpha_2 = \frac{3}{2})$ have been obtained by setting $x_1 = 0, x_2 = \frac{5(2a-5d)}{27hr^{M-25}}$)

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2 = \frac{1}{2}$</th>
<th>$\alpha_2 = 1$</th>
<th>$\alpha_2 = \frac{3}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{(2a-4)r^M(4-18hr^{M}+a(9hr^{M}+4))}{2(9hr^{M}-2)^2}, \frac{ar^M(2a-3d)}{18hr^{M}-9}, \frac{(a-4)^2r^M(9hr^{M}-2)}{(9hr^{M}-4)^2}$</td>
<td>$\frac{(-5a+9(\alpha_1-4)hr^M)(-20a+9hr^{M}(2a+4))}{2b(27hr^{M}-25)^2}, \frac{(2a-5d)r^M(-95a+125a+81hr^{M}(a-4))}{2(27hr^{M}-25)^2}$</td>
<td>$\frac{(2a-3d)r^M(3.4+\alpha(9hr^{M}-8))}{18(3hr^{M}-2)^2}, \frac{(2a-3d)r^M(3.4+\alpha(9hr^{M}-8))}{18(3hr^{M}-2)^2}$</td>
</tr>
<tr>
<td>$1$</td>
<td>$\frac{(a-4)^2r^M(9hr^{M}-2)}{(9hr^{M}-4)^2}, \frac{(a-4)^2r^M(9hr^{M}-2)}{(9hr^{M}-4)^2}$</td>
<td>$\frac{(18hr^{M-1})(8a+9hr^{M}(-2a+4))}{18b^2r^M(54hr^{M-35})^2}, \frac{(a+6hr^M(a-2d))(3.4hr^M(128-81hr^{M})+a(-32+6hr^M(81hr^{M}-68)))}{9b^2r^M(54hr^{M-35})^2}$</td>
<td>$\frac{(2a-3d)r^M(3.4+\alpha(9hr^{M}-8))}{18(3hr^{M}-2)^2}, \frac{(2a-3d)r^M(3.4+\alpha(9hr^{M}-8))}{18(3hr^{M}-2)^2}$</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: \( a=100, b=1, r=10, A=35.5, 0<\theta<1 \)
Figure 2: $a=100$, $b=1$, $r=10$, $A=5.1$, $0<\theta<1$
Figure 3: $a=1$, $b=0.1$, $r=1$, $A=0.76$, $0<\theta<1$