Optimal Vesting Conditions for Equity-Based Compensation

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Optimal Vesting Conditions for Equity-based Compensation†

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Abstract

We investigate the optimal design of equity-based compensation components, such as shares or stock options, with performance-based vesting conditions in a two-period model where a manager is hired by a firm to perform effort in two tasks. One task has an impact on short-term firm value, the other task influences the firm’s long-term value. We find that market conditions as well as firm and manager characteristics have an impact on the optimal design of equity-based compensation. There exists a substitutional relationship between the number of shares or stock options granted and the proportion of them that vests early. Moreover, complementarities between the tasks lead to a spill-over effect of long-term factors on short-term incentive provision. The common practice of installing gradual vesting schemes with equal vesting proportions, which is frequently used by firms, generally does not generate efficient incentives.

Keywords: Stock options, Vesting, Equity-based compensation, Incentives, Long-term performance

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1 Introduction

The compensation of top managers and chief executive officers (CEOs) has been and still is a widely and intensely discussed topic, both in research and practice, and is subject to huge media coverage (e.g., Vergne et al. 2018; Kuhnen and Niessen 2012). Researchers have developed a variety of theoretical models to identify optimal compensation systems. Moreover, many empirical studies have been conducted to test the implications of several compensation components such as stock or stock options, short-term and long-term incentive plans, bonus payments, etc. on manager behavior, firm performance, or investment decisions. Governments have discussed and partially implemented several regulations regarding executive compensation. Recently, the say-on-pay regulations in the US and Europe have put another dimension to the acceptability of executive compensation by shareholders.

An incentive contract is usually designed with the purpose to align the interests of the executives that manage the firm and are responsible for making decisions with those of owners or shareholders of the firm. Owners or shareholders typically have a long-term interest in the firm, whereas managers are more short-term focused, at least because they have finite professional lives. As a consequence, managers might have a higher interest in performing activities with a short-term impact on firm value compared to performing activities that secure the long-term success of the firm but do not affect the short-run firm value. As an example, managers might focus their activities on generating revenue by selling products but do not invest enough in maintenance. However, maintenance activities ensure the proper functioning of the production machinery and are thus important for the firm to be able to generate revenues in the future. In some cases, managers might have an incentive to perform
actions that have a positive impact on the firm’s short-term profit, but are even detrimental to the firm’s long-term performance. For example, current cost reduction efforts that boost short-term profitability might lead to a cut in R&D investments that are harmful for the firm’s future.

One possible approach to countervail this problem is to put top executives into a position where they become owners of the firm (e.g., Holmstrom and Milgrom 1991). From a pure incentive perspective, managers should own 100 percent of the firm. However, risk aversion and wealth constraints put a natural limit to using these incentives from equity ownership and usually prevent a solution in which managers become owners of the complete firm. One possibility to make them at least partially owners is to include equity-based components into managerial compensation, such as granting stock or stock options, and thereby link the compensation to the firm’s long-term performance which is expressed in the stock price development. In order to emphasize the long-term focus and for retention arguments, shares or stock options are usually not paid out immediately, but vest over a particular time horizon. A longer vesting period is generally associated with more farsightedly actions by the manager (e.g., Bolton et al. 2006; Peng and Röell 2014). For an even better alignment of interests, empirical research supports the use of performance-based stock or stock options, i.e., shares or stock options are vested based on the manager’s performance in a particular time period. The common belief is that performance-vested options and stocks create a greater link between the manager’s long-term oriented effort and his reward (e.g., Bettis et al. 2010; Kuang and Qin 2009).

Another problem that arises when setting incentives for top executives is that they are usually responsible for a variety of tasks, including actions that have an impact on the firm
value in the short run, but also activities that have an impact on the long-term firm value. Among some of these actions there might exist complementarities which vary with the type of business the firm operates in. For example, in a pharmaceutical company, the effort put today into the production of drugs in the firm’s current product portfolio probably helps to identify areas in which it pays off to invest long-term oriented R&D efforts. As a consequence, potential interdependencies between short-term and long-term actions should be considered in the optimal design of incentives (e.g., Dikolli et al. 2009) and the equity-based components of incentive contracts.

In total, relatively little is known on the optimal design of executive compensation with equity-based components that vest over time. The most common vesting schemes used in practice consider a vesting period of three to five years with gradual vesting, e.g., when the vesting period is four years, each year, 25 percent of the shares granted can be sold or stock options can be exercised (Hall and Murphy 2002). In the fiscal year 2011, 68 percent of S&P 500 companies granted options to their executives and 82 percent of S&P 500 companies had stock grants. The grant-date value of stock options accounted for 21 percent of median total pay for S&P 500 CEOs and 36 percent of the median total pay for stock grants (Murphy 2013). Moreover, firms increasingly make use of performance-vesting propositions: While in 1998, 21 percent of firms used performance-based vesting, in 2012, the proportion has risen to 68 percent (Bettis et al. 2018).

In this paper, we investigate the optimal design of performance-based vesting conditions for equity-based compensation components in a two-period principal-agent model in which the agent is responsible for two tasks, one with an impact on short-term firm value and one with an impact on long-term firm value. In particular, we focus on the number of shares or
stock options that should be optimally granted and on the proportion of them that should vest at each point in time.\footnote{Our model can be equally applied to both types of equity-based compensation, i.e., shares or stock options. We specify the differences between them in Section 2 where we explain the basic model.} We further investigate how potential complementarities between short-term and long-term efforts influence the design of equity-based compensation and, in particular, the vesting conditions.

We find that the optimal design of equity-based compensation depends on the stock price development, firm characteristics, and the manager’s characteristics. This corresponds to empirical findings which demonstrate that firms adapt their incentives to competition or future growth perspectives (e.g., Karuna 2007; Gaver and Gaver 1993). Moreover, the number of shares or stock options granted and the proportion of them that vests early (i.e., in period 1) are interdependent. As a consequence, a change in the total number of shares or stock options granted must be accompanied by an adjustment of the vesting conditions. More precisely, there exists a substitutional relationship, i.e., if the total number of shares or stock options increases, the proportion of shares or options that vest early should decrease.

We find that if the firm expects the value of the equity-based compensation to rise in the future, the optimal proportion of shares or stock options that vest early is higher. We also find that complementarities between short-term and long-term efforts lead to spill-over effects of long-term factors on short-term incentive provision, but not vice versa. Finally, and most interestingly, we find that the gradual vesting schemes with equal vesting proportions over the whole vesting period that are commonly used in practice are rarely optimal.

To our knowledge, the design of optimal incentive contracts with performance-based shares or stock options has been investigated very sparsely in an analytical context. This
impression is also confirmed by Bettis et al. (2010, p. 3850), who state that “... we know of no theoretical work that derives a role specifically for performance-vesting (p-v) provisions.”

Laux (2010) analyzes project decision-making when executive stock options vest if a certain stock price performance target is met. He finds that it is optimal to condition the vesting of options on stock price performance and that early vesting leads to a better project termination. Kuang and Suijs (2006) show that performance-based vesting conditions lead to higher managerial effort compared to traditional stock options when the performance target is not too high. However, they superimpose one type of performance-based vesting in a principal-agent model with only a single task. Johnson and Tian (2000) perform numerical simulations to demonstrate the superiority of performance-vesting provisions, but do not search for the optimal incentive scheme. Other theoretical research examines the optimal design of incentive contracts with executive stock options that have other vesting provisions. For example, Brisley (2006) investigates how executive stock options create risk-taking incentives and shows that early vesting can be beneficial as it improves the manager’s project selection. Closest to our paper is the analysis of Laux (2012) who investigates the optimal vesting conditions for motivating short-term and long-term activities of a CEO. The model shows that firms with more valuable long-term investment opportunities allow a larger fraction of executive stock options to vest early. The explanation is that longer vesting periods can backfire and induce excessive short-term incentives. However, Laux (2012) does not consider performance-based vesting conditions.

We contribute to the current literature by showing that the optimal vesting conditions depend on factors related to the stock price development, the manager’s, and the firm’s characteristics. Moreover, we are the first to investigate the impact of potential comple-
mentarities between short- and long-term activities on the optimal design of equity-based compensation. Our results demonstrate that commonly used practices for designing vesting conditions is not optimal for setting incentives. The model can furthermore be used as a tool for shareholders to understand executive compensation contracts and assess the adequateness of equity-based compensation. In this regard, the results provide a first idea for guiding say-on-pay decisions.

2 Basic Model

2.1 Effort Choices, Contract, and Stock Price Formation

We employ a two-period model with two risk-neutral contracting parties: A principal, acting on behalf of the firm’s owners, hires an agent (manager) to provide two productive efforts at the beginning of period 1. The agent’s first effort \( a \in (a_L, a_H) \) has a short-term impact on the firm’s outcome in period 1. In particular, the short-term effort \( a \) influences the success probability \( q \) of the firm’s short-run outcome \( x_1 \in (x_1^L, x_1^H) \), where higher effort increases the probability of a good outcome:

\[
\begin{align*}
\text{Prob}[x_1 = x_1^H | a = a_H] &= q_H, \\
\text{Prob}[x_1 = x_1^L | a = a_H] &= 1 - q_H, \\
\text{Prob}[x_1 = x_1^H | a = a_L] &= q_L, \\
\text{Prob}[x_1 = x_1^L | a = a_L] &= 1 - q_L,
\end{align*}
\]

where \( 1 > q_H > q_L > 0 \). The agent’s long-term effort \( b \in (b_L, b_H) \) has an impact on the success probability \( p \) for the firm’s long-run outcome \( x_2 \in (x_2^L, x_2^H) \). Moreover, there is an interdependency between the short-term and the long-term effort choices which is expressed
as a complementarity (e.g., Nikias et al. 2005; Zhang 2003). In particular, the probability that long-term effort \( b \) leads to a success in the second period depends on the choice of short-term effort \( a \). If the agent chooses high short-term effort \( a_H \), the success probability for the second period is positively affected by a complementarity factor \( \gamma \in (1, \frac{1}{p_H}) \). Table 1 illustrates the probabilities for long-term firm outcome depending on both effort choices. We assume that \( 1 > p_H > p_L > 0 \). The following interpretations of the complementarity factor \( \gamma \) are offered. First, \( \gamma \) can be interpreted as a factor that expresses the linkages between short-term and long-term effort. For example, in the oil production, the current oil drilling efforts are not directly linked to the future success in finding new oil springs, hence \( \gamma \) would be close to 1. For a company producing technical devices, the long-term oriented R&D efforts will benefit from experiences gained during the production process of current products, so \( \gamma \) would be rather high. Second, \( \gamma \) could represent the time lag between the impact of short-term and long-term efforts. A high \( \gamma \) means that there is a direct connection between the short-term and long-term success, which could also be applied to the technical devices company. A low \( \gamma \) would rather represent a situation where the time lag between short-term and long-term success is rather high, for example, in pharmaceutical companies, where the development of new products starts a long time period before they actually contribute to generating revenues.

<table>
<thead>
<tr>
<th>( {a, b} )</th>
<th>( x_2 = x^H_2 )</th>
<th>( x_2 = x^L_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {a_H, b_H} )</td>
<td>( \gamma \cdot p_H )</td>
<td>( (1 - \gamma \cdot p_H) )</td>
</tr>
<tr>
<td>( {a_H, b_L} )</td>
<td>( \gamma \cdot p_L )</td>
<td>( (1 - \gamma \cdot p_L) )</td>
</tr>
<tr>
<td>( {a_L, b_H} )</td>
<td>( p_H )</td>
<td>( (1 - p_H) )</td>
</tr>
<tr>
<td>( {a_L, b_L} )</td>
<td>( p_L )</td>
<td>( (1 - p_L) )</td>
</tr>
</tbody>
</table>

Table 1: Probabilities for long-term outcome
The agent’s effort choices are unobservable and associated with personal costs \( \kappa(i_H) > 0 \) and \( \kappa(i_L) = 0, i = a, b \). The principal offers the agent a contract that is composed of a fixed salary \( f \) and a variable, equity-based component such as stock or stock options with performance-based vesting conditions:\(^2\)

\[
C(f, v, n, E) = f + v \cdot n \cdot \text{Prob}[x_1 = x^H_1 | a] \cdot \max\{0, [P_1 - E]\} + (1 - v) \cdot n \cdot \gamma \cdot \text{Prob}[x_2 = x^H_2 | a, b] \cdot \max\{0, [P_2 - E]\}.
\]

The terms \( \max\{0, [P_t - E]\}, t = 1, 2 \) represent the value of the stock option, depending on the share price \( P_t \) and the exercise price \( E \). If shares are granted, the exercise price \( E \) would be equal to zero and the share price would be simply given by \( P_t, t = 1, 2.\(^3\) \) The principal determines the number of shares or stock options that are granted over both periods \( n \) and – in order to motivate the agent to provide long-term effort – designs the vesting conditions of shares or stock options as a performance-based vesting plan, that is, the agent receives the shares or stock options only if a high firm outcome \( x^H_t, t = 1, 2 \) is realized. The proportion \( v \) of shares or stock options vests early, i.e., in period 1, the proportion \( (1 - v) \) vests late, i.e., in period 2. Upon vesting, the agent owns the shares or stock options. For stock options, the following common assumptions are made: The agent is not allowed to trade or hedge the stock options, i.e., when they vest, the agent cannot sell them, but he can exercise the options (Hall and Murphy 2002). We assume that the agent exercises the stock options as

\(^2\)There exists no other more general contract that can yield a higher payoff to the principal. See the Appendix for the optimal unrestricted contract.

\(^3\)We further assume that the exercise price corresponds to the stock price in period 0 as documented by Murphy (1999) who finds empirical evidence that the exercise price is equal to the fair market value on the grant date.
soon as they are exercisable. Empirical studies show that executives exercise their options and buy the corresponding shares relatively early after they become vested for reasons of risk reduction (e.g., Huddart and Lang 1996). Furthermore, companies frequently use cashless exercise programs where executives simply receive the value of the spread between the market stock price and the exercise price in cash (Hall and Murphy 2003).

The stock price formation is based on the firm’s current and expected future performance. The firm’s expected outcome in the first period depends on the agent’s short-term effort choice \(a\) and is given by:

\[
E[x_1 | a = a_H] = q_H x^H + (1 - q_H) x^L \equiv \Pi^H_1, \text{ and } \\
E[x_1 | a = a_L] = q_L x^H + (1 - q_L) x^L \equiv \Pi^L_1.
\]

In the second period, the firm’s outcome depends on the long-term effort \(b\). Moreover, the short-term effort in the first period influences the success probability for the second period. The expected outcome in the second period is then given by:

\[
E[x_2 | a = a_H, b = b_H] = \gamma p_H x^H + (1 - \gamma p_H) x^L \equiv \Pi^{HH}_2, \\
E[x_2 | a = a_H, b = b_L] = \gamma p_L x^H + (1 - \gamma p_L) x^L \equiv \Pi^{HL}_2, \\
E[x_2 | a = a_L, b = b_H] = p_H x^H + (1 - p_H) x^L \equiv \Pi^{LH}_2, \text{ and } \\
E[x_2 | a = a_L, b = b_L] = p_L x^H + (1 - p_L) x^L \equiv \Pi^{LL}_2.
\]

\(^4\)See the literature on overlapping generations models, e.g., Banerjee (2011), Fischer et al. (2016), and Watanabe (2008).

\(^5\)For simplicity and without loss of generality, we assume that \(x^H_1 = x^H_2 = x^H\) and \(x^L_1 = x^L_2 = x^L\).
The firm’s stock price is based on rational expectations and given by\textsuperscript{6}

\[ P^j_t = \alpha_t + \beta_t \cdot \Pi^j_t, \ t = 1, 2, j = L, H \text{ if } t = 1 \text{ and } j = LL, LH, HL, HH \text{ if } t = 2. \]

The following figure depicts the timeline of events.

\begin{figure}
\begin{center}
\begin{tikzpicture}
\draw (0,0) -- (6,0);
\draw (0,0.5) -- (0.5,0.5) node[anchor=west] {t = 0};
\draw (1,0.5) -- (1.5,0.5) node[anchor=west] {t = 1};
\draw (2,0.5) -- (2.5,0.5) node[anchor=west] {t = 2};
\draw (0.5,0) -- (0.5,-0.5) node[anchor=north] {Principal offers contract to the agent};
\draw (1.5,0) -- (1.5,-0.5) node[anchor=north] {Agent exercises short-term effort a and long-term effort b};
\draw (2.5,0) -- (2.5,-0.5) node[anchor=north] {Outcome x_2 and stock price P_2 are realized};
\draw (0.25,1.5) -- (0.25,2) node[anchor=south] {At the end of t = 1 outcome x_1 and stock price P_1 are realized};
\draw (1.25,1.5) -- (1.25,2) node[anchor=south] {Agent receives proportion (1 - v) of shares or stock options if x_2 = x^H};
\draw (2.25,1.5) -- (2.25,2) node[anchor=south] {Agent receives proportion v of shares or stock options if x_1 = x^H};
\end{tikzpicture}
\end{center}
\caption{Timeline of events}
\end{figure}

2.2 Agent’s and Principal’s Preferences and First-best Benchmark

The agent is risk-neutral and protected by limited liability such that payments have to be non-negative. His utility is given by the expected compensation less personal effort costs, i.e.,

\[ U_A = E[C(f, v, n, E)] - \kappa(a) - \kappa(b). \]

\textsuperscript{6}Please refer to the Appendix for a complete derivation of the stock prices.
The agent’s reservation utility is normalized to zero. The principal maximizes the firm’s net payoff given by

$$U_P = E[x_1 | a] + E[x_2 | a, b] - E[C(f, v, n, E)].$$

For simplicity, we abstract from any discount factors and disregard the time value of money. We assume that the difference in probabilities and payoffs is large enough to cover the effort costs such that the principal always prefers to motivate high effort choices for both, short-term and long-term effort. When designing the contract, the principal needs to consider that the agent is only willing to exert high effort when the corresponding incentive compatibility constraints are fulfilled, i.e.,

$$IC_{HL} : q_H \cdot v \cdot n \cdot (P^{H}_1 - E) + \gamma p_H \cdot (1 - v) \cdot n \cdot (P^{HH}_1 - E) - \kappa(a_H) - \kappa(b_H) \geq q_H \cdot v \cdot n \cdot (P^{H}_1 - E) + \gamma p_L \cdot (1 - v) \cdot n \cdot (P^{HH}_1 - E) - \kappa(a_H),$$

$$IC_{LH} : q_H \cdot v \cdot n \cdot (P^{H}_1 - E) + \gamma p_H \cdot (1 - v) \cdot n \cdot (P^{HH}_1 - E) - \kappa(a_H) - \kappa(b_H) \geq q_L \cdot v \cdot n \cdot (P^{H}_1 - E) + p_H \cdot (1 - v) \cdot n \cdot (P^{HH}_1 - E) - \kappa(b_H),$$

$$IC_{LL} : q_H \cdot v \cdot n \cdot (P^{H}_1 - E) + \gamma p_H \cdot (1 - v) \cdot n \cdot (P^{HH}_1 - E) - \kappa(a_H) - \kappa(b_H) \geq q_L \cdot v \cdot n \cdot (P^{H}_1 - E) + p_L \cdot (1 - v) \cdot n \cdot (P^{HH}_1 - E).$$

The agent is willing to exert high effort in both tasks (i.e., $a = a_H$ and $b = b_H$) if the expected utility from exerting high efforts is larger than the expected utility from exerting any other combination of efforts.

The first-best solution to the principal’s optimization problem is provided as a benchmark.
In the first-best setting, the agent’s effort choices, \( a \) and \( b \), are observable and contractible. The principal then offers the agent a forcing contract that compensates the agent only for the effort costs and pays a fixed wage of \( f^{FB} = \frac{\kappa(a_H) + \kappa(b_H)}{2} \) in each of the two periods if high effort was chosen. The firm’s resulting first-best net payoff is given by:

\[
U^{FB}_P = q_H x^H + (1 - q_H)x^L + \gamma p_H x^H + (1 - \gamma p_H)x^L - \kappa(a_H) - \kappa(b_H).
\]

3 Optimal Contracts

If the agent’s effort choices are unobservable, the principal chooses the parameters of the contract, i.e., the vesting proportion \( v \) and the number of shares or stock options \( n \), to maximize his utility given the agent’s limited liability and incentive compatibility constraints. The agent’s participation constraint is always fulfilled given limited liability and incentive compatibility are considered.\(^7\) Which of the above three incentive compatibility constraints are binding depends on the relative expected value of the equity-based compensation component in periods 1 and 2. Consider first the case in which the relative expected value of the equity-based compensation component increases from period 1 to period 2, i.e.,

\[
\frac{q_H(P^H_1 - E)}{q_L(P^L_1 - E)} < \frac{\gamma p_H(P^H_2 - E)}{p_L(P^L_2 - E)}.
\]

In this case, the incentive compatibility constraints \( IC_{HL} \) and \( IC_{LL} \) are binding. Solving the corresponding conditions with respect to the number of shares or stock options \( n \) and the

\(^7\)The fixed wage \( f \) is always equal to zero in the optimal solution and thus omitted in the following analysis (see Laux (2012)).
vesting proportion \( v \) yields:\(^8\)

\[
\begin{align*}
n^\dagger_{\text{inc}} &= \frac{\kappa(b_H)(1-v)\gamma(p_H - p_L) \cdot P_{2HH}^H}{(q_H - q_L) \cdot P_{1HH}^H - (\gamma p_H - p_L) \cdot P_{2HH}^H} \\
v^\dagger_{\text{inc}} &= \frac{\kappa(a_H) + \kappa(b_H) - n(\gamma p_H - p_L) \cdot P_{2HH}^H}{n((q_H - q_L) \cdot P_{1HH}^H - (\gamma p_H - p_L) \cdot P_{2HH}^H)}. \tag{3}
\end{align*}
\]

The optimal vesting proportion \( v^\dagger_{\text{inc}} \) and the optimal number of shares or stock options \( n^\dagger_{\text{inc}} \) are interdependent and substitutes, i.e., if \( n \) increases, \( v \) is reduced, and vice versa. Solving the system of equations given by expressions (2) and (3) yields the optimal number of shares or stock options and the optimal vesting proportion characterized in Proposition 1.

**Proposition 1** If the relative expected value of the equity-based compensation is lower in period 1 than in period 2, the optimal compensation package is uniquely specified by the number of shares or stock options and the proportion of them that vest in period 1 with:

\[
n^\dagger_{\text{inc}} = \frac{\kappa(a_H)\gamma(p_H - p_L) P_{2HH}^H}{(q_H - q_L) P_{1HH}^H \gamma(p_H - p_L) P_{2HH}^H} + \frac{\kappa(b_H)((q_H - q_L) P_{1HH}^H - (\gamma - 1)p_L P_{2HH}^H)}{(q_H - q_L) P_{1HH}^H \gamma(p_H - p_L) P_{2HH}^H}. \tag{4}
\]

and

\[
v^\dagger_{\text{inc}} = \frac{\kappa(a_H)\gamma(p_H - p_L) - \kappa(b_H)((q_H - q_L) P_{1HH}^H - (\gamma - 1)p_L P_{2HH}^H)}{\kappa(a_H)\gamma(p_H - p_L) P_{2HH}^H + \kappa(b_H)((q_H - q_L) P_{1HH}^H - (\gamma - 1)p_L P_{2HH}^H)}. \tag{5}
\]

**Proof.** See Appendix. \( \blacksquare \)

Consider now the case in which the relative expected value of the equity-based compensation component in period 1 is higher than the relative expected value in period 2, i.e.,

\[
\frac{q_H(P_{1HH}^H - E)}{q_L(P_{1HH}^H - E)} > \frac{\gamma p_H(P_{2HH}^H - E)}{p_L(P_{2HH}^H - E)}. \]

\(^8\)For simplification and without loss of generality, we assume an exercise price equal to zero \((E = 0)\).
In this case, the incentive compatibility constraints $IC_{LH}$ and $IC_{LL}$ are binding. Solving the corresponding conditions with respect to $n$ and $v$ yields the following optimal number of shares or stock options and optimal vesting proportion, whereas the condition for the optimal vesting proportion is the same as in the previous scenario:

$$n^\dagger_{dec} = \frac{\kappa(a_H)}{v(q_H - q_L)P_1^H + (1 - v)(\gamma p_H - p_H)P_2^{HH}} \quad \text{and} \quad (6)$$

$$v^\dagger_{dec} = v^\dagger_{inc}. \quad (7)$$

Solving the system of equations given by expressions (6) and (7) yields the optimal number of shares or stock options and the optimal vesting proportion characterized in Proposition 2.

**Proposition 2** If the relative expected value of the equity-based compensation is higher in period 1 than in period 2, the optimal compensation package is uniquely specified by the number of shares or stock options and the proportion of them that vest in period 1:

$$n^\dagger_{dec} = \frac{\kappa(a_H)(p_H - p_L)P_2^{HH} + \kappa(b_H)((q_H - q_L)P_1^H - (\gamma - 1)p_H P_2^{HH})}{(q_H - q_L)P_1^H(p_H - p_L)P_2^{HH}} \quad (8)$$

and

$$v^\dagger_{dec} = \frac{\kappa(a_H)(p_H - p_L)P_2^{HH} - \kappa(b_H)(\gamma - 1)p_H P_2^{HH}}{\kappa(a_H)(p_H - p_L)P_2^{HH} + \kappa(b_H)((q_H - q_L)P_1^H - (\gamma - 1)p_H P_2^{HH})}. \quad (9)$$

**Proof.** See Appendix. ■

The optimal vesting conditions and the optimal number of shares or stock options depend on the interplay of three factors: (i) the agent’s personal characteristics in form of his effort costs, (ii) firm-specific factors such as the success probabilities for high profit, and (iii) market conditions in form of the stock price development. As the principal uses $n$ and $v$ to design an
optimal compensation package, in the following, we focus on the number of shares or stock options that vest in periods 1 and 2, which are given by

\[
n_{\text{inc}}^* \circ v_{\text{inc}}^* = \frac{\kappa(a_H)\gamma(p_H - p_L)P_{2}^{HH} - \kappa(b_H)(\gamma - 1)p_L P_{2}^{HH}}{(q_H - q_L)P_{1}^{H} \gamma(p_H - p_L)P_{2}^{HH}} \tag{10}
\]

for period 1 and by

\[
n_{\text{inc}}^* \cdot (1 - v_{\text{inc}}^*) = \frac{\kappa(b_H)}{\gamma(p_H - p_L)P_{2}^{HH}} \tag{11}
\]

for period 2 if the value of the equity-based component increases over time (Proposition 1).

The corresponding number of shares or stock options if the value of the equity-based component decreases over time (Proposition 2) is given by

\[
n_{\text{dec}}^* \circ v_{\text{dec}}^* = \frac{\kappa(a)(p_H - p_L)P_{2}^{HH} - \kappa(b)(\gamma - 1)p_H P_{2}^{HH}}{(q_H - q_L)P_{1}^{H} (p_H - p_L)P_{2}^{HH}} \tag{12}
\]

for period 1 and by

\[
n_{\text{dec}}^* \cdot (1 - v_{\text{dec}}^*) = \frac{\kappa(b)}{(p_H - p_L)P_{2}^{HH}} \tag{13}
\]

for period 2.

We first relate the optimal equity-based components to empirical findings. Karuna (2007) finds that firms provide stronger incentives when industry competition is larger. More competition in our model would, for example, be expressed by decreasing success probabilities, i.e., \(q_H\) in the first period and \(p_H\) in the second. As can be seen from equation (10), the value of incentives provided in the first period increases if \(q_H\) is decreased. For the second period, equation (11) increases when \(p_H\) is decreased. Both observations confirm the empiri-
cal observation that more competition leads to stronger incentives provided by equity-based compensation.

We next analyze the impact of the complementarity $\gamma$ on the optimal design of equity-based compensation. Motivation of short-term effort $a_H$ increases the probability of firm success in the short run directly and, through the complementarity, indirectly in the long run. In contrast, long-term effort $b$ has only an impact on the long-term firm success and does not influence the short-term outcome. If long-term effort is more important for the firm, for example, but the probability $p_H$ for long-term success is low, more shares or stock options should vest in period 2 and less in period 1 at the cost of short-term effort. However, if short-term effort is more important for the firm, this has no effect on the provision of incentives for long-term effort. Taken differently, stock options or shares vested in period 1 and period 2 are substitutes for motivating long-term effort. Corollary 1 summarizes the result.

**Corollary 1** The complementarity between short-term and long-term effort leads to a spill-over effect of long-term factors on short-term incentive provision in such a way that stock options vested in period 1 and those vested in period 2 are substitutes for motivating long-term effort.

**Proof.** See Appendix. ■

The impact of the complementarity on incentive provision further depends on the relative expected value of the equity-based component in each period. High complementarity means that the exertion of short-term effort leads to a higher probability for long-term firm success. Thus, incentives for increasing long-term firm success can be reduced. In particular, the
principal reduces the incentive component that has the higher incentive effect. In other words, when the expected value of the equity-based component in period 1 is lower than that in period 2 (corresponding to the scenario described in Proposition 1), less stock options or shares are vested in period 2, i.e., \( \frac{\partial n_{inc}(1-v_{inc})}{\partial \gamma} < 0 \). For decreasing values of the equity-based component (corresponding to the scenario described in Proposition 2), less stock options or shares are vested in period 1, i.e., \( \frac{\partial n_{dec} v_{dec}}{\partial \gamma} < 0 \). This result is summarized in Corollary 2.

**Corollary 2** Incentives for increasing long-term firm success can be reduced when effort complementarities are high. Therefore, the incentive component with the higher incentive effect is diminished.

**Proof.** See Appendix. ■

## 4 Design of Equity-based Compensation

As shown in the previous section, the optimal vesting proportions depend – among others – on the market conditions and, in particular, the stock price development. The stock price is influenced by the investors’ expectations about future firm performance, but also contains elements of noise traders. The principal must therefore account for the market conditions when designing the optimal equity-based compensation for the agent. The mechanism can be seen from comparing the optimal proportions that vest in period 1 derived in the two scenarios described in Propositions 1 and 2.

**Proposition 3** The proportion of shares or options vested in period 1 is larger if the value of the equity-based compensation increases over time, i.e., \( v_{inc}^+ > v_{dec}^+ \).
Proof. See Appendix.

The reason for the result in Proposition 3 is that in the first scenario (described in connection with Proposition 1), the expected relative value of the equity-based component in period 1 is lower than in period 2. A lower value of the equity-based compensation in the first period would lead to lower incentives for the agent to exert effort. To countervail this effect the principal vests a larger proportion of the equity-based compensation in period 1.

This result confirms the finding of Laux (2012) where firms with more valuable long-term investment opportunities allow a larger fraction of stock options to vest early. More valuable long-term investment opportunities should increase long-term firm value and thus lead to a situation in which the future value of the equity-based compensation components increases. In our model, similar circumstances allow to vest a larger proportion of shares or stock options in period 1.

Most companies offer vesting plans with equal vesting proportions of shares or options in each year during the vesting period. For example, Delta Air Lines, Inc. has a vesting plan in which executive stock options have a vesting period of three years. A proportion of $\frac{1}{3}$ of the options vests in each of these years, if the performance in the respective year corresponds to the Profit Sharing Program for the respective year.9 Our results show that these gradual vesting schemes are rarely optimal from an incentive perspective. Proposition 4 specifies a condition under which gradual vesting schemes with equal vesting proportions are optimal.10

---


10As the structure of the optimal vesting conditions and the optimal number of stock options presented in connection with Propositions 1 and 2 is similar, for the following analyses, we focus on the first case in which the value of the equity-based component increases over time. However, all of the following results can be derived from the opposed scenario as well.
Proposition 4  Gradual vesting with equal vesting proportions over the vesting period of an equity-based compensation component is optimal if, and only if,

\[
\frac{\kappa(a_H)}{\kappa(b_H)} = \frac{(q_H - q_L)P_1^H + (\gamma - 1)p_LP_2^{HH}}{\gamma(p_H - p_L)P_2^{HH}}.
\]  

(14)

Proof. See Appendix. ■

The condition presented in connection with Proposition 4 represents the case in which \( v = \frac{1}{2} \), i.e., an equal proportion of shares or options vests in both periods. This would be the particular result for our two-period model that corresponds to common corporate vesting practices. However, the condition shows that this common policy of offering gradual vesting plans with equal proportions of options or stocks that vest over time is rarely optimal. In particular, it depends on the relation of various exogenous factors. The left-hand side of equation (14) represents the relation between the agent’s effort costs for exerting short-term and long-term effort. The right-hand side shows in the numerator the expected stock price in the first and second period if high short-term effort is exerted (given long-term effort is low, i.e., \( q_HP_1^H \) and \( \gamma p_LP_2^{HH} \)) less the value if low short-term effort is exerted (i.e., \( q_LP_1^H \) and \( p_LP_2^{HH} \)). In the denominator, the expected stock price in the second period for high long-term effort \( (\gamma p_HP_2^{HH}) \) minus the expected stock price for low long-term effort \( (\gamma p_LP_2^{HH}) \), given short-term effort is high, is given. Gradual vesting is thus optimal if the relation of short-term to long-term effort costs corresponds to the relation between the increase of the stock price resulting from exerting high short-term effort and the one resulting from exerting high long-term effort. We note that all parameters in equation (14) are exogenous and characteristics of either the firm or the manager.
Assume for a moment that the firm is in a steady state in which the condition in equation (14) holds and gradual vesting is optimal. Now consider that due to some new production techniques the short-term success probability $q_H$ of the firm is increased. As a consequence, the right-hand side of equation (14) increases. However, if all other factors remain unchanged, gradual vesting is no longer optimal for the firm as the relation between effort costs does not change. One important implication of our result is thus that the observed common corporate practice in designing vesting conditions generally does not constitute an optimal design of incentives.

As a final step, we further analyze the impact of different agent characteristics on the optimal design of equity-based compensation. Short-term effort costs $\kappa(a_H)$ and short-term success probabilities $q_H$ and $q_L$ only affect the incentive provision for short-term effort. However, long-term effort costs $\kappa(b_H)$ and long-term success probabilities $p_H$ and $p_L$ have an impact on the provision of incentives for short-term and long-term effort. In particular, there exists a spill-over effect of long-term factors on short-term incentives but short-term factors do not have an impact on long-term incentives. Observation 1 details this result.

Observation 1

(a) Changes in short-term factors have an impact on the number of shares or stock options granted and vested in period 1, but leave the number of shares or stock options vested in period 2 unchanged.

(b) Changes in long-term factors have an impact on the total number of shares or stock options granted and on the optimal vesting.

If effort costs for exerting high short-term effort $\kappa(a_H)$ are high, the principal needs to
compensate the agent for these higher costs in order to motivate him to still provide high effort $a_H$. As a consequence, more stock options or shares will be vested in period 1 to compensate for higher effort costs, i.e., $\frac{\partial n^v}{\partial \kappa(a_H)} > 0$. This adjustment is realized by increasing both, the total number of options or shares, i.e., $\frac{\partial n}{\partial \kappa(a_H)} > 0$, and the proportion of the equity-based component vested in period 1, i.e., $\frac{\partial v}{\partial \kappa(a_H)} > 0$. However, it can be observed that the number of stock options or shares vested in period 2 is not affected by these circumstances. Figure 2 illustrates the total number of stock options or shares as well as the number of stock options or shares vested in period 1.

Figure 2: Impact of short-term effort cost on optimal vesting of equity-based incentive component. The parameter values are $x^H = 10$, $x^L = 0.1$, $E = 0$, $\alpha_1 = 0$, $\alpha_2 = 0$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $p_H = 0.7$, $p_L = 0.3$, $q_H = 0.7$, $q_L = 0.3$, $\kappa(b_H) = 4$, and $\gamma = 1.1$.

It is evident that the overall number of stock options or shares included in the incentive contract (solid line) increases with short-term effort costs to the same extent as the number of stock options or shares that vest in period 1 (dashed line) do. Thus, the number of options or stocks vested in period 2 (represented by the distance between the two lines) remains constant.

The picture reverses if the costs for exerting high long-term effort $\kappa(b_H)$ are considered. Although both efforts are exerted in period 1, it seems to be beneficial for the principal to
shift a larger proportion of shares or options that vest into the second period if long-term effort costs are high, i.e., $\frac{\partial n^i(1-v^i)}{\partial \kappa(b_H)} > 0$. Furthermore, less options are vested in period 1, i.e., $\frac{\partial n^0}{\partial \kappa(b_H)} < 0$. The intuition behind this result is that the principal raises the number of shares or options vested in period 2 in order to let the agent participate on the firm’s long-term success that is affected by long-term effort. Thereby, incentives for exerting long-term effort are increased though all tasks are performed (and effort costs are incurred) in period 1. However, incentives in period 1, i.e., the number of stock options or shares vested in period 1, can be reduced as more shares or options are vested in period 2. Taking a look at the individual components, the proportion of options vested in period 1 decreases with $\kappa(b_H)$, i.e., $\frac{\partial v^i}{\partial \kappa(b_H)} < 0$, whereas the reaction of the total number of stock options included in the incentive contract depends on the degree of effort complementarity, i.e., $\frac{\partial n^i}{\partial \kappa(b_H)} \neq 0$. In particular, when the degree of effort complementarity is rather low, the total number of stocks or options increases with long-term effort costs $\kappa(b_H)$, whereas the total number decreases with long-term effort costs for a high degree of complementarity. The reason is that the agent benefits from synergies resulting from high complementary tasks when performing short-term and long-term effort. Thus, total incentives can be reduced.

Figure 3 illustrates two scenarios for the development of the total number of stock options (solid line) and the number of stock options that vest in period 1 (dashed line) dependent on the costs for exerting high long-term effort.

Panel A of Figure 3 depicts the results for a low complementarity $\gamma$ between short-term and long-term effort. In this case, the total number of shares or stock options granted increases with the effort costs. However, the number of shares or options that vest early, i.e., in period 1, decreases with $\kappa(b_H)$ leading to a situation in which the corresponding number of shares
Figure 3: Impact of long-term effort cost on optimal vesting of equity-based incentive component. The parameter values are $x^H = 10$, $x^L = 0.1$, $E = 0$, $\alpha_1 = 0$, $\alpha_2 = 0$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $p_H = 0.7$, $p_L = 0.3$, $q_H = 0.7$, $q_L = 0.3$, $\kappa(a_H) = 4$, and $\gamma = 1.1$ for Panel A; $\gamma = 1.4$ for Panel B.

or options that vests late, i.e., in period 2, increases. This is illustrated by the increasing distance between the solid and dashed line in Panel A of Figure 3. Panel B of Figure 3 shows the result for a high complementarity $\gamma$ between both efforts. Then, high long-term effort costs cause the total number of total stock options to decrease. However, the number of options or stocks vested in period 2 (the distance between the two lines) also increases with effort costs $\kappa(b_H)$ whereas the number of options or shares vested in period 1 decreases with effort costs $\kappa(b_H)$.

5 Conclusion

In this paper, we investigate the optimal design of equity-based compensation components in a multi-task environment. We particularly focus on the granting of stocks or stock options with performance-based vesting conditions to a manager that is responsible for performing effort on two tasks: one with an impact on short-term firm value and one with an impact on long-term firm value. The main factors that drive the optimal design of equity-based compensation are related to (1) the market, e.g., the stock price development which depends
on the firm’s expected future outcomes but is also subject to the activities of noise traders, (2) to the manager’s characteristics, e.g., the personal effort costs, and (3) to the firm’s characteristics, e.g., the potential outcomes and the probabilities for success. Moreover, potential complementarities between both tasks lead to adjustments in the design of vesting conditions. We also investigate whether the common corporate practice of granting stocks or stock options over a vesting period \( t \) with gradual vesting of a proportion of \( \frac{1}{t} \) in each period is providing efficient incentives. Our results demonstrate that this is generally not the case. The implication from this result can be twofold: either, companies do not use equity-based compensation components for incentive purposes, but more for retention purposes, i.e., to keep the manager with the firm in the future; or, current incentive contracts simply do not generate efficient incentives and firms need to adapt them.

Our study is subject to some limitations. First, we exclusively focus on the design of equity-based compensation components. We do not investigate the interplay with potential other components of executive compensation, such as annual bonus payments, which might as well have an impact on incentive provision. Second, we investigate the optimality of equity-based compensation from an incentive perspective and do not include other factors like retention or preventing excessive risk-taking.

Our findings should help firms to identify optimal equity-based compensation structures. Moreover, they can be used as a tool by shareholders to assess the adequateness and appropriateness of manager compensation which could be useful regarding the say-on-pay decisions in the U.S. and forthcoming in Europe.
6 Appendix: Proofs

Derivation of the stock price

The stock price is constructed following the literature on overlapping generations models (e.g., Banerjee 2011, Fischer et al. 2016, Spiegel 1998, and Watanabe 2008). A continuum of homogeneous investors can invest in shares of the firm and can borrow or lend at a risk free rate, \( r > 0 \). Investors are assumed to be risk-averse whereas the contracting parties are assumed to be risk-neutral. Thus, we allow for the case in which the risk attitude concerning an investment strategy can differ from the risk attitude concerning an employment relationship. At the beginning of each period, the firm’s earnings (i.e., the terminal outcome \( \Pi_j, j = 1, 2 \)) are disclosed and completely paid out as a dividend. Investors live for two periods while the firm lives for ever. In the first period, they invest their wealth \( w \). In the subsequent period, they sell the shares and consume their wealth. An investor’s utility in the first period of life \( t \) is given by the negative-exponential function:

\[
U_t = \frac{1}{\rho} (1 - \exp[-\rho(z(\tilde{\Pi}_{t+1} + \tilde{P}_{t+1}) + (1 + r)(w - zP_t))])
\]

where \( \rho \) denotes the coefficient of the investor’s risk aversion, \( P_t \) and \( P_{t+1} \) are the share prices at date \( t \) and \( t + 1 \), and \( z \) is the quantity of shares that the investor demands. The sign ”\( \sim \)” denotes the investor’s expectation as the respective variable is not yet realized at date \( t \). Investors form their expectations about the firm’s future outcome based on the present year’s outcome, i.e.:

\[
\tilde{\Pi}_{t+1} = E[\Pi_{t+1}|\Pi_t] = \Pi_t + E[\tilde{\theta}_{t+1}|\Pi_t], \tag{15}
\]
with the variable $\tilde{\theta}_{t+1}$ as the investors expectation about the change in the firm’s outcome from date $t$ to $t + 1$. The conditional expectation is given by:

$$
E[\tilde{\theta}_{t+1}|\Pi_t] = \left(\text{prob}(\theta_{t+1}^H|\Pi_t^H) \cdot \text{prob}(\Pi_t^H) + \text{prob}(\theta_{t+1}^H|\Pi_t^L) \cdot \text{prob}(\Pi_t^L)\right)\theta_{t+1}^H + \\
\left(\text{prob}(\theta_{t+1}^L|\Pi_t^H) \cdot \text{prob}(\Pi_t^H) + \text{prob}(\theta_{t+1}^L|\Pi_t^L) \cdot \text{prob}(\Pi_t^L)\right)\theta_{t+1}^L
$$

with $\text{prob}(\theta_{t+1}^H|\Pi_t^H) = \varphi_H$, $\text{prob}(\theta_{t+1}^H|\Pi_t^L) = \varphi_L$, $\text{prob}(\theta_{t+1}^L|\Pi_t^H) = (1-\varphi_H)$, and $\text{prob}(\theta_{t+1}^L|\Pi_t^L) = (1-\varphi_L)$. The stock price can be written as follows:

$$
P_t = \alpha_t + \beta_t \cdot \Pi_t.
$$

Each investor chooses a quantity of shares to maximize his utility:

$$
\max_{z_t} z_t \cdot (\Pi_{t+1} + P_{t+1}) + (1+r)(w-z_t \cdot P_t) - \frac{\rho}{2} \cdot z_t^2 \cdot \text{Var}[\Pi_{t+1} + P_{t+1}]
$$

As the market clears, $z_t = 1$ must hold:

$$
\Rightarrow z_t^* = \frac{\Pi_{t+1} + P_{t+1} - (1+r)P_t}{\rho \cdot \text{Var}[\Pi_{t+1} + P_{t+1}]} = 1
$$

$$
\Rightarrow P_t = \frac{1}{(1+r)}(\Pi_{t+1} + P_{t+1} - \rho \cdot \text{Var}[\Pi_{t+1} + P_{t+1}])
$$
Insert equation (15):

\[ P_t = \frac{1}{(1 + r)} (\alpha_{t+1} + (1 + \beta_{t+1})E[\tilde{\theta}_{t+1} | \Pi_t] - \rho \cdot (1 + \beta_{t+1})^2 \cdot Var[\Pi_t + E[\tilde{\theta}_{t+1} | \Pi_t]]) \]

\[ + \frac{1}{(1 + r)} (1 + \beta_{t+1}) \Pi_t \]

This equation implies that an equilibrium is defined by any \( \alpha_t \) and \( \beta_t \) that satisfies the following conditions:

\[ \alpha_t = \frac{1}{(1 + r)} (\alpha_{t+1} + (1 + \beta_{t+1})E[\tilde{\theta}_{t+1} | \Pi_t] - \rho \cdot (1 + \beta_{t+1})^2 \cdot Var[\Pi_t + E[\tilde{\theta}_{t+1} | \Pi_t]]) \]

\[ \beta_t = \frac{1}{(1 + r)} (1 + \beta_{t+1}). \]

For the two periods, the following stock prices are realized:

\[ P_0 = \alpha_0 = E, \]

\[ P_{1H} = \alpha_1 + \beta_1 \cdot \Pi_{1H}, \]

\[ P_{1L} = \alpha_1 + \beta_1 \cdot \Pi_{1L}, \text{ and} \]

\[ P_{2HH} = \alpha_2 + \beta_2 \cdot \Pi_{2HH}, \]

\[ P_{2HL} = \alpha_2 + \beta_2 \cdot \Pi_{2HL}, \]

\[ P_{2LH} = \alpha_2 + \beta_2 \cdot \Pi_{2LH}, \]

\[ P_{2LL} = \alpha_2 + \beta_2 \cdot \Pi_{2LL}. \]
Optimal unrestricted contract

Consider a general contract \((w_1^H, w_1^L, w_2^{HH}, w_2^{HL}, w_2^{LH}, w_2^{LL})\) where \(w_1^H\) is the wage payment to the agent in period 1 if \(x_1 = x^H\) and \(w_1^L\) is the wage if \(x_1 = x^L\), \(w_2^{il}\) is the payment in period 2 if \(x_1 = x^i\) and \(x_2 = x^l, i, l = H, L\). We assume without loss of generality that \(w_1^L = w_2^{LL} = 0\). The agent’s utility is given by

\[
U_A = E[C(w_1^H, w_1^L, w_2^{HH}, w_2^{HL}, w_2^{LH}, w_2^{LL})] - \kappa(a) - \kappa(b).
\]

The principal maximizes the firm’s net payoff given by

\[
U_P = E[x_1 | a] + E[x_2 | a, b] - E[C(w_1^H, w_1^L, w_2^H, w_2^L)].
\]

The incentive compatibility constraints are as follows:

\[
IC_{HL} : \quad q_H \cdot w_1^H + q_H(\gamma p_H \cdot w_2^{HH} + (1 - \gamma p_H) \cdot w_2^{HL}) + (1 - q_H)\gamma p_H \cdot w_2^{LH} - \kappa(a_H) - \kappa(b_H) \\
\geq q_H \cdot w_1^H + q_H(\gamma p_L \cdot w_2^{HH} + (1 - \gamma p_L) \cdot w_2^{HL}) + (1 - q_H)\gamma p_L \cdot w_2^{LH} - \kappa(a_H),
\]

\[
IC_{LH} : \quad q_H \cdot w_1^H + q_H(\gamma p_H \cdot w_2^{HH} + (1 - \gamma p_H) \cdot w_2^{HL}) + (1 - q_H)\gamma p_H \cdot w_2^{LH} - \kappa(a_H) - \kappa(b_H) \\
\geq q_L \cdot w_1^H + q_L(\gamma p_H \cdot w_2^{HH} + (1 - p_H) \cdot w_2^{HL}) + (1 - q_L)p_H \cdot w_2^{LH} - \kappa(b_H),
\]

\[
IC_{LL} : \quad q_H \cdot w_1^H + q_H(\gamma p_H \cdot w_2^{HH} + (1 - \gamma p_H) \cdot w_2^{HL}) + (1 - q_H)\gamma p_H \cdot w_2^{LH} - \kappa(a_H) - \kappa(b_H) \\
\geq q_L \cdot w_1^H + q_L(\gamma p_L \cdot w_2^{HH} + (1 - p_L) \cdot w_2^{HL}) + (1 - q_L)p_L \cdot w_2^{LH}.
\]

For determining the optimal contract, the incentive compatibility constraints are binding. Solving the equation system with these constraints for \(w_2^{HH}, w_2^{HL}, \text{and } w_2^{LH}\) leads to the
optimal wage payments in dependence of $w_{1H}$. The optimal wage payments are inserted in
the function of the firm’s net payoff where $w_{1H}$ has no further impact. The resulting net
payoff is lower than in the model with the stock-based compensation component.

Proof. Proof of Proposition 1

For determining the optimal contract when the relative expected value of compensation
increases from period 1 to period 2, the incentive compatibility constraints $IC_{HL}$ and $IC_{LL}$
are binding. When these constraints are binding, incentive compatibility constraint $IC_{LH}$
is fulfilled. Inserting equation (3) into (2) and solving for $n$ leads to the optimal number of
shares or stock options presented in equation (4). Inserting equation (4) into (3) yields the
optimal vesting proportion given in equation (5).

Proof. Proof of Proposition 2

For determining the optimal contract when the relative expected value of compensation
decreases from period 1 to period 2, the incentive compatibility constraints $IC_{LH}$ and $IC_{LL}$
are binding. The derivation of the optimal contract is analogous to the proof of Proposition 1.

The principal offers the contract from Proposition 1 when the relative expected value of
compensation increases from period 1 to period 2, i.e., under condition (1), as the firm’s net
payoff is higher with this contract than with the contract from Proposition 2. When the
relative expected value of compensation decreases over time, the firm’s net payoff is higher
with the contract from Proposition 2 so that she offers that contract to the agent.
Proof. Proof of Corollary 1

The following table illustrates the spill-over effect of long-term factors on short-term incentive provision:

Panel A: Impact of short-term factors on the optimal number of shares or stock options vested in period 1 and 2

<table>
<thead>
<tr>
<th>Parameter j</th>
<th>$sgn[\partial (n_{inc}^\dagger \cdot v_{inc}^\dagger)/\partial j]$</th>
<th>$sgn[\partial (n_{inc}^\dagger \cdot (1 - v_{inc}^\dagger))/\partial j]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa(a_H)$</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$q_H$</td>
<td>$sgn[\frac{\kappa(b_H)(\gamma-1)p_L P_H^H - \kappa(a_H)}{\gamma(p_H-p_L) P_H^H}]$</td>
<td>0</td>
</tr>
<tr>
<td>$q_L$</td>
<td>$sgn[\kappa(a_H) - \frac{\kappa(b_H)(\gamma-1)p_L P_H^H}{\gamma(p_H-p_L) P_H^H}]$</td>
<td>0</td>
</tr>
</tbody>
</table>

Panel B: Impact of long-term factors on the optimal number of shares or stock options vested in period 1 and 2

<table>
<thead>
<tr>
<th>Parameter j</th>
<th>$sgn[\partial (n_{inc}^\dagger \cdot v_{inc}^\dagger)/\partial j]$</th>
<th>$sgn[\partial (n_{inc}^\dagger \cdot (1 - v_{inc}^\dagger))/\partial j]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa(b_H)$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$p_H$</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$p_L$</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2: Comparative statics of the optimal number of shares or stock options in each period

The derivatives with respect to short-term factors (Panel A) indicate that changes in short-term factors have no impact on the long-term incentive provision. In contrast, changes in long-term factors (Panel B) affect the optimal incentive provision in both periods. More precisely, if more options or shares vest in period 2, for example, due to lower long-term success probability, less options or shares are vested in period 1. Thus, stock options and shares vested in period 1 and 2 are substitutes for motivating long-term effort. ■
Proof. Proof of Corollary 2

The impact of effort complementarity on the number of stock options or shares vested in period 2, when the relative expected value of compensation increases from period 1 to period 2, is illustrated by differentiating equation (11) with respect to $\gamma$:

$$\frac{\partial (n_{inc}^+ \cdot (1 - v_{inc}^+))}{\partial \gamma} = -\frac{\kappa(b_H)(\gamma p_{HH}^2 + \gamma p_H(x^H - x^L))}{(p_H - p_L)\gamma^2(P_{HH}^2)^2} < 0.$$ 

As all parameters are positive and the success probability is higher when the agent chooses a high effort level, i.e., $p_H > p_L$, and $x^H > x^L$, the above derivative is negative. Thus, a high effort complementarity $\gamma$ leads to less shares or stock options vested in period 2.

The impact of effort complementarity on the number of stock options vested in period 1, when the relative expected value of compensation decreases from period 1 to period 2, is illustrated by differentiating equation (12) with respect to $\gamma$:

$$\frac{\partial (n_{dec}^+ \cdot v_{dec}^+)}{\partial \gamma} = -\frac{\kappa(b)p_H}{(q_H - q_L)P_{HH}^1(p_H - p_L)} < 0.$$ 

As all parameters are positive and the success probabilities are higher when the agent chooses a high effort level, i.e., $p_H > p_L$, $q_H > q_L$, the above derivative is negative. Thus, a high effort complementarity $\gamma$ leads to less shares or stock options vested in period 1. ■
Proof. Proof of Proposition 3

To prove the statement we compare the optimal vesting proportions given in equations (5) and (9). We obtain the proof by formulating the opposite statement and then showing that a contradiction with general assumptions will result.

\[ v_{inc}^* < v_{dec}^* \text{ if} \]

\[
\left[ \kappa(a_H)\gamma(p_H - p_L)P^{HH}_2 - \kappa(b_H)(\gamma - 1)p_LP^{HH}_2 \right] \cdot \\
\left[ \kappa(a_H)(p_H - p_L)P^{HH}_2 + \kappa(b_H)((q_H - q_L)P^H_1 - (\gamma - 1)p_HP^{HH}_2) \right] < \\
\left[ \kappa(a_H)(p_H - p_L)P^{HH}_2 - \kappa(b_H)(\gamma - 1)p_LP^{HH}_2 \right] \cdot \\
\left[ \kappa(a_H)(\gamma p_H - \gamma p_L)P^{HH}_2 + \kappa(b_H)((q_H - q_L)P^H_1 - (\gamma - 1)p_LP^{HH}_2) \right] \\
\Leftrightarrow \kappa(b_H)(q_H - q_L)P^H_1 \cdot \left[ \kappa(a_H)\gamma(p_H - p_L) - \kappa(b_H)(\gamma - 1)p_L \right] < \\
\kappa(b_H)(q_H - q_L)P^H_1 \cdot \left[ \kappa(a_H)(p_H - p_L) - \kappa(b_H)(\gamma - 1)p_H \right] \\
\Leftrightarrow \left( \kappa(a_H) + \kappa(b_H) \right) \left[ (\gamma p_H - \gamma p_L) - (p_H - p_L) \right] < 0 \\
\Leftrightarrow (\gamma - 1)p_H < (\gamma - 1)p_L \\
\Leftrightarrow p_H < p_L
\]

As this condition is contradictory to the assumption \( p_H > p_L \), \( v_{inc}^* > v_{dec}^* \) is always satisfied.
Proof. Proof of Proposition 4

In the two-period model, gradual vesting with equal vesting proportions would mean that in each period, 50% of the shares or stock options vest. We thus derive the condition based on the requirement that the vesting proportion in the first period is \( v_{inc} = \frac{1}{2} \) (in case of increasing expected values of compensation). Then:

\[
\frac{k(a_H)\gamma(p_H - p_L)P_{2HH}^{HH} - k(b_H)(\gamma - 1)p_L P_{2HH}^{HH}}{k(a_H)\gamma(p_H - p_L)P_{2HH}^{HH} + k(b_H)((q_H - q_L)P_{2HH}^{H} - (\gamma - 1)p_L P_{2HH}^{HH})} = \frac{1}{2}
\]

\[
\Leftrightarrow 2 \cdot \left[ k(a_H)\gamma(p_H - p_L)P_{2HH}^{HH} - k(b_H)(\gamma - 1)p_L P_{2HH}^{HH} \right]
\]

\[
= k(a_H)\gamma(p_H - p_L)P_{2HH}^{HH} + k(b_H)((q_H - q_L)P_{2HH}^{H} - (\gamma - 1)p_L P_{2HH}^{HH})
\]

\[
\Leftrightarrow k(a_H)\gamma(p_H - p_L)P_{2HH}^{HH} = k(b_H)((q_H - q_L)P_{2HH}^{H} + (\gamma - 1)p_L P_{2HH}^{HH})
\]

\[
\Leftrightarrow \frac{k(a_H)}{k(b_H)} = \frac{(q_H - q_L)P_{1H}^{H} + (\gamma - 1)p_L P_{2HH}^{HH}}{\gamma(p_H - p_L)P_{2HH}^{HH}}.
\]

The proof would be analogous for the setting with decreasing expected values of compensation, i.e., for \( v_{dec} = \frac{1}{2} \). \( \blacksquare \)
References


Murphy Kevin J 2013. Executive compensation: Where we are, and how we got there Handbook of the Economics of Finance 2: 211–356.


