Cost Uncertainty and the Welfare Effects of Market Power

Marco de Pinto & Laszlo Goerke

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Cost Uncertainty and the Welfare Effects of Market Power

Marco de Pinto∗
IAAEU and Trier University

Laszlo Goerke†
IAAEU and Trier University, IZA Bonn and CESifo München

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Abstract

How does uncertainty about production costs affect the welfare consequences of an oligopoly? To answer this question, we set up a model of a homogenous Cournot oligopoly with free but costly entry. Production costs are uncertain ex-ante, i.e. before entering the market. We show that the excessive entry distortion will be aggravated by cost uncertainty if entry is sufficiently costly and, hence, the market not too competitive. In addition, the positive gap between the socially optimal and the equilibrium expected output per firm and the welfare loss due to market power increase. This suggests a heightened importance of competition authorities in a world characterized by uncertainty.

Keywords: Oligopoly, Excessive Entry, Uncertainty, Welfare

JEL Classification: D 43, L 13
1 Introduction

Oligopolies are pervasive. This is true for many consumer products, such as electronic devices, cars, insurance contracts, airline traveling, food, beverages, and also for many inputs. Head and Spencer (2017) accordingly conclude that "oligopoly is a robust characteristic of a broad set of industries in the US and around the world." In addition, markets have become more oligopolistic over time. Autor et al. (2017), for example, show that 4-firm concentration ratios have increased in the major sectors of the United States economy over the last thirty years. These findings suggest that also the detrimental welfare consequences due to market power have become greater over time. Such conclusion implicitly assumes that an oligopolistic market structure is the only deviation from the textbook case of a perfectly competitive market. Accordingly, an important task of economic research is to gain knowledge of how other deviations from the textbook case, which is often used to derive policy advice and to determine welfare effects, interact with the distortion due to market power.

In this regard, one widely unexplored feature is that firms face uncertainty when deciding whether or not to start production, that is, whether or not to enter a market. Such uncertainty can have a multitude of causes, e.g. unknown prices, per unit costs or marginal productivities. We focus on the impact of productivity or, which is analytically equivalent, marginal costs. A prominent example for this type of uncertainty is the digitization process, which has become a key feature of nearly all industrialized countries. Due to the rise of automation and artificial intelligence, many firms are confronted with uncertainty about productivity levels. For instance, McKinsey&Company (2017) argue that digitization creates performance and productivity opportunities, but reshapes also current business models with the risk of substantial profit reductions. As such, not only market power has increased over time but also cost uncertainty.\footnote{There are many examples of how digitization has influenced uncertainty. Whereas two decades ago the costs of producing a car were relatively predictable, nowadays driver-less cars using other energy sources than gasoline may constitute the future. Clearly, the costs of producing such a car are highly unpredictable. A firm employing the ‘wrong’ technology will then face higher production costs than a firm that can utilize a more productive combination of inputs. Similarly, the administrative costs associated with the sale of credit and insurance contracts have declined in recent years because human involvement has been replaced by algorithm-based decisions. The extent of cost savings, once again, depends on which technology is employed and is unknown when the technology choice is made, i.e. when the entry decision is taken.}
In this paper, we therefore investigate how cost uncertainty affects the welfare implications of oligopoly. In the short-run, oligopolies generally reduce welfare because of their impact on output. In the long-run, when entry and exit of firms are taken into account, market entry decisions are likely to be inefficient, as well. This is because a firm ignores the consequences of entry for the payoff of its competitors. If output per firm declines with their number, such business-stealing externality may give rise to excessive entry. Allowing for cost uncertainty can substantially alter these effects because it affects a firm’s expected profits and, therefore, the net gain from entry.

To analyze whether cost uncertainty aggravates or mitigates the adverse welfare consequences due to market power, we set up a model of a homogenous Cournot oligopoly in which firms first decide about entry. Entry requires an investment. It allows firms to draw a realization of productivity (or cost) from an according distribution. While the distribution is known, the actual realization of costs is revealed only after entry has taken place, but before production decisions are made. Hence, firms are identical ex-ante but differ ex-post. Uncertainty is modeled as a variation in unit marginal production costs. Since such uncertainty affects the position and slope of the supply curve, it has been labeled multiplicative uncertainty in earlier contributions. This distinguishes it from additive uncertainty which affects only the slope of supply (or demand) schedules.

In this setting, we first investigate how cost (or multiplicative) uncertainty affects output decisions and the number of competitors in market equilibrium. We show that the incentives to enter the market rise if entry costs exceed a critical level. This is the case because firms can save on production costs. Moreover, in oligopoly each competitor partially takes into account the impact of its quantity choice on the output price. Hence, the increase in aggregate output and the decline in price are relatively moderate. Expected profits then rise with cost uncertainty. In consequence, it seems as if cost uncertainty aggravates the excess entry outcome occurring in oligopolistic markets.

Before drawing such a conclusion, however, one has to regard that uncertainty also changes the optimal number of firms. This is (in a second-best world) determined by a social planner who takes into account quantity decisions by firms. We demonstrate that if the costs of market entry are sufficiently high, also the optimal number of firms rises
with cost uncertainty. As the social planner chooses a lower number of firms than there will be in market equilibrium, each firm produces a higher quantity. Incurring additional entry due to cost uncertainty becomes attractive for the social planner for a higher level of fixed costs than that amount which ensures more firms in market equilibrium. Thus, we have three possible outcomes: If entry costs are relatively low, cost uncertainty will reduce both the equilibrium and optimal number of firms. If entry costs are sufficiently high, the reverse will be true. For intermediate levels, the equilibrium number of firms will rise, whereas the optimal number will decline.

We finally demonstrate, using numerical simulations, that cost uncertainty raises the number of excessive entrants, irrespective of the (relative) level of entry costs, and increases the positive difference between expected output per firm in the social optimum and equilibrium. Foremost, cost uncertainty raises the overall welfare loss due to oligopoly. These predictions have important policy conclusions. They suggest that the potential welfare gains from regulating oligopolies are larger in markets characterized by uncertainty. Given an increase in uncertainty, the role of competition authorities in a world featuring globalization, digitization and also increasing political uncertainty becomes more important.

In the remainder of the paper we initially survey related contributions in Section 2. We outline the model in Section 3 and determine the market equilibrium and the (second-best) optimal outcome in Section 4. In Section 5, we first show that cost uncertainty never neutralizes the oligopoly distortion. Second, we analyze how cost uncertainty affects the number of firms, output and welfare in market equilibrium and in a setting in which a social planner determines entry. Subsequently, we compare the changes. Section 6 provides some concluding remarks.

2 Literature Review

Our paper is most closely related to two strands of literature. The first analyzes the profit and welfare consequences of price and cost uncertainty in competitive markets. The second investigates the determinants of excessive entry and resulting welfare distortions
in oligopoly. In addition, our study has similarities to several papers in which firms do not know their productivity (or cost) when deciding whether or not to enter a market.

The effects of price variability on the payoffs of consumers and firms have initially been investigated by Waugh (1944) and Oi (1961) and been combined in a unified framework by Massell (1969). These authors considered shifts in the demand or supply curve in a competitive market and showed that consumers or firms can benefit from such (additive) price variability to which they can respond by adjusting behavior. Massell (1969) demonstrated that the sum of expected consumer surplus and profits decreases with price instability. Turnovsky (1976) subsequently clarified that the detrimental welfare effects carry over to settings in which price variability also alters the slope of demand and supply curve because it affects prices and costs multiplicatively. These studies were often inspired by the empirically observable price variations in agricultural products. The ensuing question was whether governments should use buffer stocks to reduce price variability over time.

Our analysis assumes that firms face (multiplicative) cost uncertainty of a different type. More specifically, some firms face high while others experience low costs at a given point in time. This assumption captures the idea that cost uncertainty arises because firms are heterogeneous ex-post. Moreover, it implies that consumer surplus is constant for a given extent of cost variability. Ex-post heterogeneity is empirically a more relevant feature than cost homogeneity, even in a market for a homogeneous product. Moreover, our setting is consistent with the view that firms’ responses to changes in economic conditions vary because their management or workforce have differential abilities to cope with the resulting uncertainty. Hence, our investigation reflects some key properties of the in-

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2There is also a strand of literature that studies the welfare distortions in markets with monopolistic competition. The pioneering work in this field is the paper by Dixit and Stiglitz (1977). The authors show that the business-stealing effect interacts with a love-of-variety effect, which could lead to insufficient entry. There are many extensions of this basic framework, e.g. by incorporating heterogeneous firms (see Dhingra and Morrow, 2018, Zhelobodko et al., 2012) or by considering heterogeneous sectors (Behrens et al., 2018).

3This approach differs from settings in which the realization of the price can only be observed after output decisions have been made and in which firms are risk-averse (see Baron, 1970, Leland, 1972, Sandmo, 1971). The effects of uncertainty on entry decision have also been looked at in contributions utilizing the real options approach (see, e.g., Dixit, 1989). In this setting, uncertainty alters the value of waiting such that it affects the timing and possibly volume of market entry investments.

4Accordingly, we do not have to worry about the appropriate welfare measure relating to the payoff of consumers in the presence of uncertainty (Turnovsky et al., 1980).
dustrial production process. This is partially in contrast to the analyses in the tradition of Oi (1961), Waugh (1944) and Turnovsky (1976) which are more relevant for agricultural products or natural resources and effectively assume price homogeneity at a given point in time and heterogeneity over time.

The excessive entry prediction for a homogenous Cournot oligopoly in the presence of business stealing has been established by Mankiw and Whinston (1986), Perry (1984), Suzumura and Kiyono (1987), Varian (1985), von Weizsäcker (1980); see Polo (2018) for a literature review. Follow-up investigations have confirmed the robustness of the excessive entry theorem.\(^5\) In these and most subsequent contributions, entry decision are governed by profits, while uncertainty does not play a role.

There are two exceptions: Creane (2007) assumes that firms incur entry costs but then only manage to enter the market with an exogenous probability. All actual entrants are identical. He shows that if the failure probability is high enough, insufficient entry may occur. In Deo and Corbett (2009) the production process is characterized by yield uncertainty, that is, the actual output level cannot precisely be inferred from the amount of inputs chosen. Moreover, input quantities have to be determined before the uncertainty is resolved and marginal costs are linear in inputs. They show that the number of entrants in market equilibrium is first increasing and then declining in uncertainty, aggregate expected output is likely to fall, while consumer surplus surely decreases with uncertainty. Moreover, there will be insufficient entry if uncertainty, as measured by the variance of output, exceeds a critical value. This type of uncertainty investigated by Deo and Corbett (2009) differs from what we analyze below in two dimensions. First, it does not affect productivity or costs but the actual output level by all firms. Second, firms cannot respond to the realization of the price.

In our setting, cost uncertainty effectively changes marginal production costs. Therefore, Corchón and Fradera (2002)’s finding that lower variable costs tend to raise the number of firms, output per firm and aggregate output predicts the change in market

\(^5\)For instance, Amir et al. (2014) show that the excessive entry theorem holds if the production process is characterized by increasing returns to scale, while Wang (2016) proves its robustness in an open economy setting. Moreover, in case of imperfect input markets, the business-stealing effect prevails and leads to excessive entry in many (but not all) considered settings (see, inter alia, de Pinto and Goerke, 2016, Ghosh and Morita, 2007a,b).
equilibrium to which our results can be compared. In addition, the present analysis can be related to contributions which focus on cost reductions in a costly but free-entry oligopoly (see, inter alia, Chao et al., 2017, Haruna and Goel, 2011, Mukherjee, 2012, Okuno-Fujiwara and Suzumura, 1993). In these contributions, excessive entry may no longer result and the inefficiencies often depend on the extent of ex-ante cost asymmetries and of knowledge spillovers. Both aspects play no role in our analysis of ex-ante symmetric firms. Moreover, we consider changes in the market outcome and the socially optimal situation and compare them. Previous contributions have not undertaken such assessment of variations in output and welfare.

As described above, our modeling strategy implies that firms are homogenous ex-ante, i.e. before market entry, and heterogeneous ex-post. As such, our paper has similarities to theoretical frameworks in which initially identical firms have to bear costs in order to reveal their technology. In a pioneering work, Melitz (2003) develops a model in which entrepreneurs incur costs in order to draw a productivity level from a given distribution. In many subsequent studies, for instance by Helpman et al. (2004), it is assumed that productivities are Pareto distributed and entry costs are interpreted as investments in research and development. Hence, the same investment might lead to a technology that is characterized by a relatively high or low productivity, respectively low or high unit costs. The outcome of the production process is uncertain ex-ante. Our paper differs in, at least, two dimensions from this strand of literature. First, we analyze how uncertainty affects the distortions of market power, which is not looked at in the aforementioned studies. Second, market inefficiencies arise from an oligopoly in our study while analyses in the tradition of Melitz (2003) focus on monopolistic competition.

3 Model

We consider an oligopolistic market in which \( n \) firms produce a homogenous good. Firms compete in quantities \( q \) and take the output choices of competitors as given (Cournot-Nash setting). They have to bear entry costs \( k (> 0) \), which are sunk. Costs \( k \) can be interpreted as investment which has to be undertaken to set up a production site but
yields an uncertain payoff, e.g. research and development costs. More specifically, at the time when the investment decision is made, it is known that half of the firms will be highly productive, low-cost plants, whereas the other half will be less productive, high-cost plants. Hence, unit marginal production costs will either be $c_h = c + \epsilon$ or $c_l = c - \epsilon$, $0 \leq \epsilon < c$, and the probability of either cost realization will be 50%. We interpret $\epsilon$ as a measure of ex-ante uncertainty about productivity or production costs. Once the entry decision has been made, marginal production costs are revealed.

Production costs of firm $j$, $j = 1, ..., n$, are strictly convex in output, $q_{ji}$, and given by $0.5c_iq_{ji}^2$, $i = h, l$. The inverse demand function is linear and reads $p(Q) = 1 - Q$, where $Q = q_{ji} + Q_{-j}$ equals aggregate output. Profits of firm $j$ in state $i$ can then be expressed as

$$\pi_{ji} = (1 - Q)q_{ji} - 0.5c_iq_{ji}^2 - k.$$  \hfill (1)

Aggregate output equals $Q = 0.5n(q_{jl} + q_{jh})$. Note that $Q$ varies with $\epsilon$, but there is no uncertainty with respect to the level of $Q$ since $\epsilon$ is known. Expected profits read

$$\pi_j^e = 0.5(\pi_{jl} + \pi_{jh})$$
$$= 0.5((1 - Q)(q_{jl} + q_{jh}) - 0.5c_lq_{jl}^2 - 0.5c_hq_{jh}^2) - k,$$  \hfill (2)

while welfare $W$ is defined as the sum of aggregate expected profits and consumer surplus

$$W = n\pi^e + 0.5Q^2.$$  \hfill (3)

When calculating the number of firms, $n$, we distinguish two scenarios. In the market equilibrium, expected profits determine $n$, while in the second-best optimum, a social planner selects $n$ to maximize welfare.\textsuperscript{6} The timing is as follows:

1a) Firms decide about market entry, i.e., about whether to invest $k$ or to abstain from doing so.

1b) The social planner determines the number of firms and each entrant invests $k$.

2) Marginal production costs are revealed.

3) Firms simultaneously choose output.

We solve the aforementioned game by backward induction and focus on pure strategy equilibria. Because a firm’s behavior is qualitatively the same, irrespective of how the number of competitors is determined, the analysis in 3), i.e. regarding the choice of output, applies both to the market equilibrium and the social planner’s considerations. Note that we ignore the integer constraint and treat the number of firms as a continuous variable (see, inter alia, Delipalla and Keen, 1992, Seade, 1980).

4 Solution

4.1 Output Choice

At stage 3, firms maximize profits with respect to output, taking as given the number of firms, $n$, and marginal (unit) production costs, $c_i$. The first-order condition for firm $j$ reads

$$\frac{d\pi_{ji}}{dq_{ji}} = -q_{ji} + 1 - Q - c_i q_{ji} = 0. \tag{4}$$

The second-order condition is fulfilled.

We can solve (4) for the reaction functions of firms of each type. Because all firms facing the same costs behave identically, we obtain $q_l = (1 - 0.5n q_h)/(1 + 0.5n + c_l)$ and $q_h = (1 - 0.5n q_l)/(1 + 0.5n + c_h)$. Combining these equations yields the output of each firm type for any number of firms $n$

$$q_l(\epsilon, n) = \frac{1 + c + \epsilon}{(1 + n + c)(1 + c) - \epsilon^2}, \tag{5}$$

$$q_h(\epsilon, n) = \frac{1 + c - \epsilon}{(1 + n + c)(1 + c) - \epsilon^2}, \tag{6}$$

where the denominator is surely positive because of $\epsilon < c$. 


Using (5) and (6), we can calculate expected output per firm as

$$\hat{q}(\epsilon, n) = 0.5(q_l + q_h) = \frac{1 + c}{(1 + n + c)(1 + c) - c^2}. \quad (7)$$

Since expected output per firm decreases in the number of competitors, $n$, there is business stealing. From (7), we can determine aggregate output as a function of $n$ and the uncertainty parameter $\epsilon$

$$Q(\epsilon, n) = \frac{n(1 + c)}{(1 + n + c)(1 + c) - c^2}. \quad (8)$$

### 4.2 Market Equilibrium

To determine the market equilibrium, we first compute the equilibrium number of firms $n^*$. Rearranging (2) yields

$$\pi^e(\epsilon, n) = 0.5 \left( (1 - Q^*(\epsilon, n))(q_l^*(\epsilon, n) + q_h^*(\epsilon, n)) - 0.5c_l q_l^*(\epsilon, n)^2 - 0.5c_h q_h^*(\epsilon, n)^2 \right) - k$$

$$= \frac{(1 + 0.5c)(1 + c)^2 - 0.5c\epsilon^2}{((1 + n + c)(1 + c) - c^2)^2} - k. \quad (9)$$

Inserting (9) into the free-entry condition $\pi^e(n^*, \epsilon) = 0$ and solving with respect to $n^*$ yields the number of firms in market equilibrium

$$n^*(\epsilon) = \frac{(1 + 0.5c)(1 + c)^2 - 0.5c\epsilon^2)^{0.5} k^{-0.5} + \epsilon^2 - (1 + c)^2}{1 + c}. \quad (10)$$

Given $n^*(\epsilon)$, we can calculate equilibrium output per firm, $q^*_l(\epsilon)$ and $q^*_h(\epsilon)$, by inserting (10) into (5) and (6), respectively. Plugging (10) into (7) and (8) yields the expected output per firm in equilibrium, $\hat{q}^*(\epsilon)$ (see Appendix A.1), and aggregate output

$$Q^*(\epsilon) = 1 - ((1 + c)^2 - \epsilon^2) \left( \frac{k}{(1 + 0.5c)(1 + c)^2 - 0.5c\epsilon^2} \right)^{0.5}, \quad (11)$$

with $Q^*(\epsilon) \in (0, 1)$. Welfare reads $W^*(\epsilon) = 0.5Q^*(\epsilon)^2$. 

9
4.3 Social Optimum

When selecting the welfare-maximizing number of firms, the social planner takes into account the output decisions of firms. The optimization problem is then given by

$$\max_n W(n) = n\pi^e(n, \epsilon) + 0.5Q(n, \epsilon)^2.$$  

Inserting (8) and (9), the first-order condition of that problem can be expressed as

$$\frac{dW}{dn} = \pi^e(n) + n \frac{d\pi^e}{dn} + Q(n) \frac{dQ}{dn} \begin{cases} <0 \\ >0 \end{cases}$$

$$= \frac{1}{((1+n+c)(1+c)-\epsilon^2)^2} M(\epsilon, n) = 0,$$

with

$$M(\epsilon, n) = (1+0.5c)(1+c)^2 - 0.5\epsilon c^2 - ((1+n+c)(1+c)-\epsilon^2)^2 k - \frac{n(1+c)((1+c)^2+\epsilon^2)}{(1+n+c)(1+c)-\epsilon^2}.$$  

Since $M(\epsilon, n)$ declines with $n$, the second-order condition is fulfilled.

Eq. (12) implicitly determines the number of firms in the social optimum as a function of $\epsilon$, that is $n^{opt}(\epsilon)$. The socially optimal levels of expected output per firm, aggregate output, expected profits and welfare are then given by $\tilde{q}^{opt}(\epsilon, n^{opt}(\epsilon))$, $Q^{opt}(\epsilon, n^{opt}(\epsilon))$, $\pi^{eopt}(\epsilon, n^{opt}(\epsilon))$ and $W^{opt}(\epsilon, n^{opt}(\epsilon))$, respectively (see Appendix A.1).

5 The Effects of Cost Uncertainty

If there is business stealing, entry is excessive and output per firm is inefficiently low (see, e.g., Mankiw and Whinston, 1986). This excessive entry theorem, does, however, not take uncertainty into account. In a first step, we show that even if distortions are mitigated by cost uncertainty, the effects will never be strong enough to invalidate the excess entry theorem. Second, we analyze how the endogenous variables change with cost uncertainty in market equilibrium and in social optimum and, more importantly, consider their relative variation. We will show that cost uncertainty actually aggravates
the adverse welfare consequences due to market power.

5.1 Excessive Entry Theorem and Cost Uncertainty

Comparing the number of firms, expected output per firm and aggregate output in the market equilibrium and social optimum, we find:

Proposition 1

The excessive entry theorem also holds in the presence of cost uncertainty, i.e. for \( \epsilon > 0 \).

Specifically, entry is excessive, \( n^*(\epsilon) > n^{opt}(\epsilon) \ \forall \epsilon \), expected output per firm is too low, \( \tilde{q}^*(\epsilon) < \tilde{q}^{opt}(\epsilon) \ \forall \epsilon \), and aggregate output is too high, \( Q^*(\epsilon) > Q^{opt}(\epsilon) \ \forall \epsilon \).

Proof 1

Rearranging (13) implies

\[
M(n^{opt}) = \pi^e(n^{opt}) \left( (1 + n^{opt} + c)(1 + c) - \epsilon^2 \right)^2 - \frac{n^{opt}(1 + c) ((1 + c)^2 + \epsilon^2)}{(1 + n^{opt} + c)(1 + c) - \epsilon^2} = 0. \tag{14}
\]

Evaluating Eq. (14) at \( n^* \) and noting that \( \pi^e(n^*) = 0 \), we find that \( M(n^*) < 0 \). This implies \( n^* > n^{opt} \ \forall \epsilon \). Using (7) and (8) proves the remainder of the Proposition.

Intuitively, a firm entering the market does not internalize the business-stealing externality, while the social planner takes this externality into account. In the presence of cost uncertainty, expected output per firm declines with the number of competitors. Therefore, the output externality continues to exist. Effectively, cost uncertainty does not constitute another externality which could reverse the business-stealing effect.

A comparison of Proposition 1 and the findings by Deo and Corbett (2009) indicates that the type of uncertainty we consider differs fundamentally from the variety they look at. They analyze yield uncertainty, i.e., a situation in which a given level of inputs leads to an ex-ante unknown quantitative yield and, thus, sales volume. Such uncertainty appears to alter the business-stealing externality and their findings show that it can reverse its impact. This is not true with (multiplicative) cost uncertainty.
5.2 Comparative Static Analysis

Subsequently, we present a comparative static analysis of the equilibrium and socially optimal outcomes. Considering the market equilibrium, we find:

**Proposition 2**

\[
\begin{align*}
    k_{\text{crit}}(\epsilon) &= \frac{c}{4((1 + n^*(\epsilon) + c)(1 + c) - \epsilon^2)} = \frac{(0.25c)^2}{(1 + 0.5c)(1 + c)^2 - 0.5c}\epsilon^2.
\end{align*}
\]

An increase in uncertainty, i.e. a rise in \(\epsilon\),

(i) raises (reduces) the number of firms \(n^*(\epsilon)\) if \(k > k_{\text{crit}}(\epsilon)\) \((k < k_{\text{crit}}(\epsilon))\), and

(ii) raises expected output per firm \(\bar{q}^*(\epsilon)\), total output \(Q^*(\epsilon)\) and welfare \(W^*(\epsilon)\).

**Proof 2** See Appendix A.2.

An increase in \(\epsilon\) has two effects on expected profits. First, expected output per firm increases. This is because the increase in production of the low-cost firm overcompensates the decrease in output of the high-cost firm. This cost effect raises \(\pi^e\). Note that this mechanism is also present in a competitive setting (see Appendix A.3 for the proof) and basically determines the result by Oi (1961). Second, if average output per firm rises, aggregate production increases. This reduces the equilibrium price and such price effect implies that \(\pi^e\), c.p., declines.

Which of these effects dominates crucially depends on the number of firms. If the market would be perfectly competitive, the number of firms would be relatively high. Each firm would adjust output, without taking into account that it thereby contributes to a price change. Consequently, the price variation would be relatively large in a competitive market characterized by a downward sloping demand schedule, and the price effect would dominate the cost impact. In consequence, expected profits would decline (see Appendix A.3). In our oligopoly setting, the number of firms is relatively large if market entry costs are low \((k < k_{\text{crit}}(\epsilon))\). In this scenario, market power of firms is relatively weak, i.e. oligopoly outcomes are 'close' to the results under perfect competition, which implies that expected profits decline in \(\epsilon\). Thus, the price effect continues to outweigh the cost impact. Consequently, the incentives to enter the market are reduced and \(n^*\) decreases.
If, however, entry costs are sufficiently high \((k > k^{crit}(\epsilon)}\), relatively few firms compete and each of them has a relatively large market share. This mitigates the incentives to expand output if costs are low \((c_i = c_l)\) and to reduce output if costs are high \((c_i = c_h)\) because the resulting price adjustments are larger than if each firm had a smaller market share. Consequently, in an oligopoly with 'few' competitors, the cost effect dominates the price impact. Thus, expected profits increase and the number of firms rises.

Regarding \(\tilde{q}^*\), we have already described the positive direct cost effect of an increase in \(\epsilon\). Because expected output declines with the number of firms due to the business-stealing effect, there is an indirect impact of uncertainty, too. As long as higher uncertainty leads to lower market entry, both effects go in the same direction. If entry costs are sufficiently large, \(n^*\) rises, which, c.p., reduces \(\tilde{q}^*\). This countervailing indirect force is, however, not strong enough to dominate the direct effect. The same line of argument applies to aggregate output \(Q^*\).\(^7\) The increase in \(Q^*\) leads then to an unambigious rise of welfare \(W^*\).

Based on Proposition 2, one might conjecture that the welfare loss due to a free-entry oligopoly may be lower in situations of cost uncertainty. This expectation, however, disregards the fact that the social planner will also change its choice relating to the number of firms. We summarize the alterations due to cost uncertainty in the socially optimal outcome in

**Proposition 3**

Let

\[
k^{critopt}(\epsilon) = \frac{c}{4((1 + n^{opt}(\epsilon) + c)(1 + c) - \epsilon^2)} + \frac{n^{opt}(\epsilon)^2(1 + c)^2 + 4n^{opt}(\epsilon)(1 + c)^3}{4((1 + n^{opt}(\epsilon) + c)(1 + c) - \epsilon^2)^3} > k^{crit}(\epsilon).
\]

An increase in uncertainty, i.e. a rise in \(\epsilon\),

(i) raises (reduces) the number of firms \(n^{opt}(\epsilon)\) if \(k > k^{critopt}(\epsilon)\) \((k < k^{critopt}(\epsilon))\),

(ii) raises expected output per firm \(\tilde{q}^{opt}(\epsilon)\) if \(k \leq k^{critopt}(\epsilon)\).

\(^7\)Comparing these findings with those by Corchón and Fradera (2002) indicates that the consequences of cost uncertainty of the type considered here are qualitatively the same as those resulting from a reduction in marginal costs.
(iii) raises total output $Q^{opt}(\epsilon)$ if $k \geq k^{critopt}(\epsilon)$, and

(iv) raises welfare $W^{opt}(\epsilon)$.

Proof 3 See Appendix A.4.

The second-best optimal number of firms results from the trade-off between higher market entry costs and more intensive competition and, hence, higher output. Due to the cost effect, an increase in uncertainty raises output per firm and in aggregate, for a given number of firms. This reduces the social planner’s gain from more entry. However, each entrant will produce a greater quantity. This quantity effect, in turn, raises the gain from entry. Note that when analyzing the consequences of uncertainty the price effect which is crucial in market equilibrium is replaced by what we label the quantity effect when focusing on the social optimum. This is because the social planner takes into account the implications on output and is concerned with the price only insofar as it determines the firms’ quantity choices.

If the number of firms is low, since market entry costs are high, each additional entrant produces a relatively large extra quantity. Therefore, the quantity effect dominates the cost impact and the optimal number of firms rises. In this case, the change in expected output per firm becomes indeterminate because there is a positive direct effect and an output-reducing impact of more competitors. Since total output rises with cost uncertainty and the number of firms, it will surely increase if more entry is permitted. If, in contrast, the number of firms is high, since market entry costs are low, the cost effect dominates and the social planner allows fewer firms to enter. Then, the overall output variation becomes ambiguous.

Irrespective of the direction of the change in total output, welfare rises. This is the case because its variation results from the direct impact of cost uncertainty only, given that the number of firms is chosen optimally by the social planner. The direct impact on output and consumer surplus is positive, whereas the immediate consequences for expected profits are ambiguous, given the decline in the price. However, consumer surplus rises by more with cost uncertainty than profits may decline. Thus welfare increases.
The last feature of Proposition 3 to be explained is the value of $k_{\text{critopt}}(\epsilon)$. This critical value of market entry costs, which ensures that the socially optimal number of firms, $n_{\text{opt}}(\epsilon)$, goes up, is higher than the respective value guaranteeing that $n^*(\epsilon)$ increases, i.e. $k_{\text{critopt}}(\epsilon) > k^*(\epsilon)$. This is the case since the social planner takes into account the impact on expected profits and consumer surplus, whereas only the effect on expected profits $\pi^e(\epsilon)$ matters in market equilibrium. Accordingly, the quantity effect, which is relevant for the social planner, is c.p. greater than the price impact, effective in market equilibrium. For the number of firms to rise, therefore, the cost impact in social optimum needs to be stronger than in market equilibrium. Because a smaller number of competitors implies that each of them produces a greater quantity, the cost effect will be pronounced the fewer firms there are. Having fewer competitors in social optimum is tantamount to market entry costs exceeding a higher critical level than in market equilibrium.

Comparing Propositions 2 and 3, we can note that either the variables are predicted to change similarly with cost uncertainty in market equilibrium and in a socially optimal situation or the direction of the variation cannot be predicted. There is one exception relating to the number of firms.

**Corollary 1**

Assume $k_{\text{crit}}(\epsilon) < k < k_{\text{critopt}}(\epsilon)$. An increase in $\epsilon$ raises $n^*(\epsilon) - n_{\text{opt}}(\epsilon) > 0$, i.e. makes excessive entry more pronounced.

If $k_{\text{crit}}(\epsilon) < k < k_{\text{critopt}}(\epsilon)$ holds, expected profits will rise with cost uncertainty such that the number of firms in market equilibrium increases. However, the increase in profits will not be sufficient to compensate the higher social costs of entry. Hence, the socially optimal number of firms declines and excessive entry becomes more pronounced.

### 5.3 Numerical Solution

Because the changes in output per firm and in aggregate, as well as welfare in market equilibrium, relative to the variations in a socially optimal situation, cannot be signed analytically, we subsequently resort to a numerical evaluation. This enables us to determine the relative effects of an increase in $\epsilon$ and to ascertain if the difference between market
outcome and socially optimal benchmark rises or falls.

To solve our model numerically, we set \( c = 0.9 \), implying that \( 0 \leq \epsilon < 0.9 \). This specification implies that the price-cost margin varies from about 53% for \( \epsilon = 0 \) to about 39% for \( \epsilon \to c \).\(^8\) These values are well in line with the empirical evidence. Dobbelaere (2004), for instance, uses Belgian manufacturing data and find that the price-cost margin ranges from 20% to 49%. Similar values for other countries have been estimated by Crépon et al. (2005), Abraham et al. (2009) and De Loecker and Eeckhout (2017). Lower or higher unit marginal costs than \( c = 0.9 \) match the estimated price-cost margin less accurately. In Appendix A.5, we nonetheless show that our main results, as detailed below, are not sensitive to the choice of \( c \).

We compute the differences between the socially optimal and the equilibrium outcomes, e.g. \( W^{opt}(\epsilon) - W^{*}(\epsilon) \), at \( \epsilon = 0 \) and at \( \epsilon \to c \). By calculating the percentage changes for the endogenous variables, we can quantify the effect of uncertainty on the inefficiencies in an oligopoly. We distinguish three scenarios:

I. Market entry costs \( k \) are relatively low and fall short of \( k^{crit*}(\epsilon) \).

II. Market entry costs \( k \) are moderate and lie between \( k^{crit*}(\epsilon) \) and \( k^{opt*}(\epsilon) \).

III. \( k \) is high and exceeds \( k^{opt*}(\epsilon) \).

Because \( k^{crit*}(\epsilon) \) and \( k^{opt*}(\epsilon) \) are increasing in \( \epsilon \), we choose the following parameter setting for entry costs. In scenario I, we assume \( k_{low} = 0.2 k^{crit*}(0) = 0.008 \). Likewise, we set \( k_{high} = 1.2 k^{opt*}(\epsilon) = 0.15 \) in scenario III. For scenario II, \( k_{int} = 0.08 \approx (k_{high} + k_{low})/2 \) holds.

Table 1, lines 1 and 2, show that the number of firms in market equilibrium and in the socially optimal situation declines with cost uncertainty if \( k \) is low (\( k_{low} < k^{crit*} < k^{opt*} \)) and rises if \( k \) is sufficiently high (\( k_{high} > k^{opt*} > k^{crit*} \)). For intermediate values of \( k \) (\( k^{crit*} < k_{int} < k^{opt*} \)), the number of firms in market equilibrium rises, while the socially optimal number falls. This illustrates parts (i) of Propositions 2 and 3. Line 3 of Table 1 also highlights Corollary 1 for scenario II. Interestingly, we observe that the number of excessive entrants increases irrespective of the value of market entry costs, \( k \).

\(^8\)The price-cost margin is given by \( (p(Q) - c\bar{q})/p(Q) \)
Table 1: The Effects of Uncertainty

<table>
<thead>
<tr>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{\text{low}} &lt; k_{\text{crit}}^* &lt; k_{\text{critopt}})</td>
<td>(k_{\text{crit}}^* &lt; k_{\text{int}} &lt; k_{\text{critopt}})</td>
<td>(k_{\text{crit}} &lt; k_{\text{critopt}} &lt; k_{\text{high}})</td>
</tr>
<tr>
<td>(\Delta n^*)</td>
<td>-0.49</td>
<td>11.7</td>
</tr>
<tr>
<td>(\Delta n^{opt})</td>
<td>-23.5</td>
<td>-4.7</td>
</tr>
<tr>
<td>(\Delta (n^* - n^{opt}))</td>
<td>37.1</td>
<td>47.5</td>
</tr>
<tr>
<td>(\Delta q^*)</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>(\Delta q^{opt})</td>
<td>30.3</td>
<td>16.4</td>
</tr>
<tr>
<td>(\Delta (q^{opt} - q^*))</td>
<td>85.1</td>
<td>77.9</td>
</tr>
<tr>
<td>(\Delta Q^*)</td>
<td>3.1</td>
<td>15.7</td>
</tr>
<tr>
<td>(\Delta Q^{opt})</td>
<td>-0.42</td>
<td>10.9</td>
</tr>
<tr>
<td>(\Delta (Q^* - Q^{opt}))</td>
<td>44.4</td>
<td>38.8</td>
</tr>
<tr>
<td>(\Delta \pi^{opt})</td>
<td>106.1</td>
<td>83.4</td>
</tr>
<tr>
<td>(\Delta W^*)</td>
<td>6.3</td>
<td>33.8</td>
</tr>
<tr>
<td>(\Delta W^{opt})</td>
<td>9.6</td>
<td>41.9</td>
</tr>
<tr>
<td>(\Delta (W^{opt} - W^*))</td>
<td>107.4</td>
<td>139.5.6</td>
</tr>
</tbody>
</table>

Note: \(\Delta\) indicates percentage changes of the respective variable.

Table 1, furthermore, shows that cost uncertainty makes the insufficiency of expected output per firm more pronounced because the optimal level rises substantially. Moreover, aggregate output in market equilibrium becomes more excessive with cost uncertainty. The last lines of Table 1 illustrate that welfare rises with cost uncertainty. More importantly, the welfare loss due to an oligopoly with costly, but unrestricted entry increases with cost uncertainty and is largest if entry costs, \(k\), are highest, \(k_{\text{high}} > k_{\text{critopt}}\). High entry costs imply few entrants. Therefore, cost uncertainty appears to aggravate the relative welfare loss due to market power more in the presence of few firms.

6 Conclusion

Modern economies are, inter alia, characterized by two stylized facts: a.) growing prevalence of oligopolistic market structures and b.) rising uncertainty about production outcomes e.g. due to digitization. Market power reduces welfare to below the first-best level. It is, however, not well understood how uncertainty affects this inefficiency. Therefore, in this paper we analyze the effects of ex-ante cost uncertainty on the welfare implications of oligopoly.
To that end, we consider a homogenous Cournot oligopoly model with costly market entry. Firms do not know about their unit marginal costs before they enter the market; it can be either low or high. After entry, unit marginal costs are realized and firms decide about output. We distinguish two scenarios. In the market equilibrium, the number of firms is determined by the free-entry condition. In the social optimum, a social planner sets the number of firms in order to maximize welfare, taking the output decisions of firms into account, i.e. we consider a second-best social optimum. We find that an increase in uncertainty aggravates the excessive entry distortion and raises the positive gap between the socially optimal and the equilibrium (expected) output per firm. Therefore, the welfare loss due to market power increases if outcomes of the production process become more uncertain.

This finding has a number of policy implications. First, it highlights that policies which aim to regulate oligopolies (or try to avoid their formation) are an important aspect and may become more relevant from a welfare perspective, the less predictable market success is. Second, such policies have to evaluate the market environment carefully, in particular with respect to the level of uncertainty. Third, while digitization has its advantages, our investigation points out a further drawback. Since digitization makes the firm’s profitability more uncertain, the welfare loss in oligopolistic markets becomes higher. Here, it could be necessary to compensate potential losers. It would be, however, surely the wrong way to slow down the digitization process because then the economy would, inter alia, lose (international) competitiveness and, more generally, forego the welfare gains of an encompassing rise in productivity.
A Appendix

A.1 Full Solution

A.1.1 Equilibrium

Inserting \( n^*(\epsilon) \), as defined by (10), into (5) and (6) yields the equilibrium production of a low and high cost firm

\[
q_l^*(\epsilon) = \frac{1 + c + \epsilon}{(1 + n^*(\epsilon) + c)(1 + c) - \epsilon^2},
\]

(A.1)

\[
q_h^*(\epsilon) = \frac{1 + c - \epsilon}{(1 + n^*(\epsilon) + c)(1 + c) - \epsilon^2},
\]

(A.2)

respectively. Expected output per firm in equilibrium can then be computed as

\[
\tilde{q}^*(\epsilon) = 0.5(q_l^*(\epsilon) + q_h^*(\epsilon)) = \frac{1 + c}{2(1 + c)k^{0.5}}
\]

(A.3)

Aggregate output in equilibrium is determined by inserting (10) into (8) and reads

\[
Q^*(\epsilon) = \frac{n^*(\epsilon)(1 + c)}{(1 + n^*(\epsilon) + c)(1 + c) - \epsilon^2}
\]

(A.4)

\[
= 1 - \frac{1}{((1 + 0.5c)(1 + c)^2 - 0.5ce^2)^{0.5}}.
\]

A.1.2 Social Optimum

The socially optimal number of firms, \( n^{opt}(\epsilon) \), is implicitly determined by (12). Because we consider a second-best social optimum, output per firm (for any given \( n \)) is determined by (5) and (6). Expected output per firm and aggregate output are then given by (7) and
Therefore, the respective values in the social optimum can be expressed as

\[
q_l^{\text{opt}}(\epsilon) = \frac{1 + c + \epsilon}{(1 + n^{\text{opt}}(\epsilon) + c)(1 + c) - \epsilon^2}, \quad (A.5)
\]

\[
q_h^{\text{opt}}(\epsilon) = \frac{1 + c - \epsilon}{(1 + n^{\text{opt}}(\epsilon) + c)(1 + c) - \epsilon^2}, \quad (A.6)
\]

\[
q^{\text{opt}}(\epsilon, n^{\text{opt}}(\epsilon)) = \frac{1 + c}{(1 + n^{\text{opt}}(\epsilon) + c)(1 + c) - \epsilon^2}, \quad (A.7)
\]

\[
Q^{\text{opt}}(\epsilon) = n^{\text{opt}}(\epsilon) \frac{1 + c}{(1 + n^{\text{opt}}(\epsilon) + c)(1 + c) - \epsilon^2}. \quad (A.8)
\]

Expected profits in the social optimum are given by

\[
\pi^{\text{eopt}}(\epsilon) = \frac{(1 + 0.5c)(1 + c)^2 - 0.5c^2}{((1 + n^{\text{opt}}(\epsilon) + c)(1 + c) - \epsilon^2)^2} - k, \quad (A.9)
\]

and welfare reads

\[
W^{\text{opt}}(\epsilon) = n^{\text{opt}}(\epsilon) \pi^{\text{eopt}}(\epsilon) + 0.5Q^{\text{opt}}(\epsilon)^2. \quad (A.10)
\]

**A.2 Derivation of Proposition 2**

From (9) it is obvious that \(\pi^e\) declines with \(n\). Differentiating (9) with respect to \(\epsilon\) yields

\[
\frac{d\pi^e}{d\epsilon} = -\frac{\epsilon}{((1 + n^*(\epsilon) + c)(1 + c) - \epsilon^2)^2} \left(c - 4k((1 + n^*(\epsilon) + c)(1 + c) - \epsilon^2)\right), \quad (A.11)
\]

where we have used the fact that \(\pi^e(n^*) = 0\). This implies

\[
\frac{d\pi^e}{d\epsilon} > 0 \Leftrightarrow k > \frac{c}{4((1 + n^*(\epsilon) + c)(1 + c) - \epsilon^2)} \equiv k^{\text{crit}^*}(\epsilon). \quad (A.12)
\]

By inserting (10) we can explicitly calculate the critical level of market entry costs.

Differentiating (A.3) yields

\[
\frac{dq^*}{d\epsilon} = \frac{\partial q^*}{\partial \epsilon} + \frac{\partial q^*}{\partial n^*} \frac{dn^*}{d\epsilon}, \quad (A.13)
\]

where

\[
\frac{\partial q^*}{\partial \epsilon} > 0 \quad \text{and} \quad \frac{\partial q^*}{\partial n^*} < 0
\]

\[
= \frac{0.5ceq^*}{(1 + 0.5c)(1 + c)^2 - 0.5ce^2} > 0.
\]
Differentiating (A.4) yields

\[
\frac{dQ^*}{d\epsilon} = \frac{\partial Q^*}{\partial \epsilon} + \frac{\partial Q^*}{\partial n^*} \frac{dn^*}{d\epsilon} > 0
\]

\[
= 2\epsilon k^{0.5} \left(1 + 0.25c\right)\left(1 + c\right)^2 - 0.25c\epsilon^2 \left((1 + 0.5c)(1 + c)^2 - 0.5c\epsilon^2\right)^{1.5} > 0.
\]

Because of \(W^* = 0.5Q^e^2\), this implies that \(dW^*/d\epsilon > 0\).

### A.3 A Benchmark Case: Competitive Market

To provide a better understanding of the role of inefficiencies caused by market power, we consider a benchmark case where the goods market is perfectly competitive. We assume that the number of firms is relatively large and fixed. Moreover, a firm’s output decision does not affect the market price and entry costs \(k\) play no role.

Suppose that the price \(p\) is exogenously given. Then, profit maximization of firm \(j\) implies that \(q_{ji} = p/c_i\). Assuming symmetric behavior, expected profits can be calculated as

\[
\pi^e(\epsilon) = 0.5 \left(\frac{p^2}{c_l} + \frac{p^2}{c_h} - 0.5c_l \left(\frac{p}{c_l}\right)^2 - 0.5c_h \left(\frac{p}{c_h}\right)^2\right)
\]

\[
= \frac{p^2c}{2(c^2 - \epsilon^2)}.
\]

This shows that \(d\pi^e/d\epsilon > 0\) holds.

Suppose now the price is given by the (inverse) linear demand function \(p = 1 - Q\), with \(Q = 0.5n(q_{jl} + q_{jh})\). Then, profit maximization, taking the market price as given, implies that \(1 - Q - c_i q_{ji} = 0\) and therefore

\[
q_h = \frac{1 - 0.5nq_l}{0.5n + c_h}, \quad (A.16)
\]

\[
q_l = \frac{1 + 0.5nq_h}{0.5n + c_l}. \quad (A.17)
\]
Combining (A.16) and (A.17) and assuming symmetric behavior yields

\[ q_h(\epsilon) = \frac{c - \epsilon}{(n + c)c - \epsilon^2}, \quad (A.18) \]

\[ q_l(\epsilon) = \frac{c + \epsilon}{(n + c)c - \epsilon^2}. \quad (A.19) \]

Expected profits can thus be written as

\[ \pi^e(\epsilon) = 0.5 \left( (1 - Q)(q_h + q_l) - 0.5c_lq_l^2 - 0.5c_hq_h^2 \right) \]

\[ = 0.5 \left( (1 - 0.5n(q_l + q_h))(q_h + q_l) - 0.5c_lq_l^2 - 0.5c_hq_h^2 \right) \]

\[ = \frac{0.5c(c^2 - \epsilon^2)}{(n + c)c - \epsilon^2)^2}. \quad (A.20) \]

Differentiation implies

\[ \frac{d\pi^e}{d\epsilon} = \frac{ce}{(n + c)c - \epsilon^2} \frac{(c - n)c - \epsilon^2}{(n + c)c - \epsilon^2}. \quad (A.21) \]

This shows that \( d\pi^e/d\epsilon < 0 \) holds if \( nc > c^2 - \epsilon^2 \) which is a reasonable result because the number of firms should be relatively large in a market with perfect competition.

### A.4 Proof of Proposition 3

Totally differentiating (12) implies that

\[ \frac{dn^{opt}}{d\epsilon} = -\frac{\partial M/\partial \epsilon}{\partial M/\partial n}. \quad (A.22) \]

where \( \partial M/\partial n < 0 \) holds due to the second-order condition for a maximum of \( W \). Furthermore, we obtain:

\[ \frac{\partial M}{\partial \epsilon} = \epsilon \left( 4 \left( (1 + n + c)(1 + c) - \epsilon^2 \right) k - c - \frac{2n^2(1 + c)^2 + 4n(1 + c)^3}{(1 + n + c)(1 + c) - \epsilon^2)^2} \right). \quad (A.23) \]
This implies
\[
\frac{dn^{opt}}{d\epsilon} > 0 \iff \frac{c}{4((1 + n^{opt}(\epsilon) + c)(1 + c) - \epsilon^2)} + \frac{n^{opt}(\epsilon)^2(1 + c)^2 + 4n^{opt}(\epsilon)(1 + c)^3}{4((1 + n^{opt}(\epsilon) + c)(1 + c) - \epsilon^2)^3} \\
\equiv k_{crit^{opt}}(\epsilon),
\] (A.24)

Comparing the critical values \(k_{crit}(\epsilon)\) and \(k_{crit^{opt}}(\epsilon)\), we find that \(k_{crit^{opt}}(\epsilon) > k_{crit}(\epsilon)\) because \(n^* > n^{opt}\) such that the first term in (A.24) already exceeds \(k_{crit}(\epsilon)\).

Differentiating (A.7) yields
\[
\frac{d\tilde{q}^{opt}}{d\epsilon} = \frac{\partial \tilde{q}^{opt}}{\partial \epsilon} + \frac{\partial \tilde{q}^{opt}}{\partial n^{opt}} \frac{dn^{opt}}{d\epsilon}. \quad (A.25)
\]

For \(k \leq k_{crit^{opt}}(\epsilon)\), we have \(dn^{opt}/d\epsilon \leq 0\) and, therefore, \(d\tilde{q}^{opt}/d\epsilon > 0\). For \(k > k_{crit^{opt}}(\epsilon)\), the sign of \(d\tilde{q}^{opt}/d\epsilon\) remains unclear. Differentiating (A.8) yields
\[
\frac{dQ^{opt}}{d\epsilon} = \frac{\partial Q^{opt}}{\partial \epsilon} + \frac{\partial Q^{opt}}{\partial n^{opt}} \frac{dn^{opt}}{d\epsilon} \quad \text{with} \quad (A.26)
\]
\[
\frac{\partial Q^{opt}}{\partial \epsilon} = \frac{2\epsilon n^{opt}(1 + c)}{(1 + n^{opt}(\epsilon) + c)(1 + c) - \epsilon^2)^2} > 0. \quad (A.27)
\]

For \(k \geq k_{crit^{opt}}(\epsilon)\), we have \(dn^{opt}/d\epsilon \geq 0\) and therefore \(dQ^{opt}/d\epsilon > 0\). For \(k < k_{crit^{opt}}(\epsilon)\), we have \(dn^{opt}/d\epsilon < 0\) such that the sign of \(dQ^{opt}/d\epsilon\) remains unclear.

Regarding expected profits, we obtain
\[
\frac{d\pi^{opt}}{d\epsilon} = \frac{\partial \pi^{opt}}{\partial \epsilon} + \frac{\partial \pi^{opt}}{\partial n^{opt}} \frac{dn^{opt}}{d\epsilon} \quad \text{with} \quad (A.28)
\]
\[
\frac{\partial \pi^{opt}}{\partial \epsilon} = \frac{\epsilon ((4 + c)(1 + c)^2 - c(1 + c)n^{opt} - c\epsilon^2)}{((1 + n^{opt}(\epsilon) + c)(1 + c) - \epsilon^2)^3}. \quad (A.29)
\]

Because the sign of the partial effect is unclear, the total impact of an increasing \(\epsilon\) on
\( \pi^{opt} \) is theoretically ambiguous. Finally, the effect on welfare is given by

\[
\frac{dW^{opt}}{d\epsilon} = \frac{\partial W^{opt}}{\partial \epsilon} + \frac{\partial W^{opt}}{\partial n^{opt}} \frac{dn^{opt}}{d\epsilon} = 0
\]

(A.30)

Inserting (A.27) and (A.29) yields after some rearrangements

\[
\frac{dW^{opt}}{d\epsilon} = \epsilon n^{opt} \frac{(4 + c)(1 + c)^2 - \epsilon c^2 + (2 + c)(1 + c)n^{opt}}{((1 + n^{opt} + c)(1 + c) - \epsilon^2)^3} > 0,
\]

(A.31)

because \((4 + c)(1 + c)^2 - \epsilon c^2 > 0\) holds.

### A.5 Robustness Checks

We have calculated the percentage changes of the endogenous variables owing to a variation of \( \epsilon \) from \( \epsilon = 0 \) to \( \epsilon \rightarrow c \) for lower \((c = 0.7)\) and higher \((c = 1.1)\) values of unit marginal costs, \( c \), than considered in the main text (Table 1). The main findings, as observed in Table 1, continue to hold. This is also true for other values of \( c \) (results are available upon request).
### Table 2: Lower Unit Marginal Costs ($c = 0.7$)

<table>
<thead>
<tr>
<th></th>
<th>Scenario I ($k_{low} &lt; k_{crit} &lt; k_{critopt}$)</th>
<th>Scenario II ($k_{crit} &lt; k_{int} &lt; k_{critopt}$)</th>
<th>Scenario III ($k_{crit} &lt; k_{critopt} &lt; k_{high}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta n^*$</td>
<td>-0.3</td>
<td>8.4</td>
<td>18.2</td>
</tr>
<tr>
<td>$\Delta n^{opt}$</td>
<td>-19.9</td>
<td>-4.2</td>
<td>6.4</td>
</tr>
<tr>
<td>$\Delta(n^* - n^{opt})$</td>
<td>26.9</td>
<td>33.1</td>
<td>44.3</td>
</tr>
<tr>
<td>$\Delta q^*$</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>$\Delta q^{opt}$</td>
<td>23.9</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>$\Delta(q^{opt} - q^*)$</td>
<td>60.8</td>
<td>52.4</td>
<td>62.3</td>
</tr>
<tr>
<td>$\Delta Q^*$</td>
<td>1.9</td>
<td>10.8</td>
<td>20.9</td>
</tr>
<tr>
<td>$\Delta Q^{opt}$</td>
<td>-0.7</td>
<td>7.3</td>
<td>17</td>
</tr>
<tr>
<td>$\Delta(Q^* - Q^{opt})$</td>
<td>34.1</td>
<td>27.1</td>
<td>35.3</td>
</tr>
<tr>
<td>$\Delta \pi^{eopt}$</td>
<td>77.8</td>
<td>57</td>
<td>65.1</td>
</tr>
<tr>
<td>$\Delta W^*$</td>
<td>3.9</td>
<td>22.7</td>
<td>46.1</td>
</tr>
<tr>
<td>$\Delta W^{opt}$</td>
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<td>28.6</td>
<td>54.1</td>
</tr>
<tr>
<td>$\Delta(W^{opt} - W^*)$</td>
<td>73.1</td>
<td>89.8</td>
<td>122.4</td>
</tr>
</tbody>
</table>

Note: $\Delta$ indicates percentage changes of the respective variable. The calculated values of entry costs are $k_{low} = 0.2k^{crit}(0) = 0.006, k_{int} = 0.08 \approx (k_{high} + k_{low})/2$ and $k_{high} = 1.2k^{critopt}(c) = 0.16$. 
Table 3: Higher Unit Marginal Costs ($c = 1.1$)

<table>
<thead>
<tr>
<th></th>
<th>Scenario I ($k_{low} &lt; k^{crit} &lt; k^{critopt}$)</th>
<th>Scenario II ($k^{crit} &lt; k_{int} &lt; k^{critopt}$)</th>
<th>Scenario III ($k^{crit} &lt; k^{critopt} &lt; k_{high}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta n^*$</td>
<td>-0.7</td>
<td>15.1</td>
<td>36.8</td>
</tr>
<tr>
<td>$\Delta n^{opt}$</td>
<td>-26.5</td>
<td>-4.7</td>
<td>16.2</td>
</tr>
<tr>
<td>$\Delta (n^* - n^{opt})$</td>
<td>47.5</td>
<td>63.2</td>
<td>91.9</td>
</tr>
<tr>
<td>$\Delta \tilde{q}^*$</td>
<td>5.1</td>
<td>5.1</td>
<td>5.1</td>
</tr>
<tr>
<td>$\Delta \tilde{q}^{opt}$</td>
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<td>20.7</td>
<td>17.5</td>
</tr>
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<td>107.2</td>
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<td>$\Delta (Q^* - Q^{opt})$</td>
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<td>112.9</td>
<td>139.5</td>
</tr>
<tr>
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<tr>
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<td>200.8</td>
<td>314.7</td>
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</tbody>
</table>

Note: $\Delta$ indicates percentage changes of the respective variable. The calculated values of entry costs are $k_{low} = 0.2k^{crit}(0) = 0.009$, $k_{int} = 0.08 \approx (k_{high} + k_{low})/2$ and $k_{high} = 1.2k^{critopt}(c) = 0.15$. 


References


