Bank Runs and Accounting for Illiquid Bank Assets

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Discussion Paper No. 18-15
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September 4, 2018

Abstract

We consider a bank that finances long-term loans with short term deposits. At the rollover stage, the depositors observe a forward looking signal and decide whether to roll over or withdraw their deposits. Withdrawals must be financed by liquidating loans in an illiquid market. We consider the interaction between the bank’s choice of deposit contracts and the information system. In line with the earlier literature, an aggressive information system that does not trigger false alarms induces efficient deposit contracting. We consider the case where such a system is not available and seek for alternatives. We identify a second deposit contract where the bank voluntarily offers rents to the depositors. These rents are socially beneficial because they reduce the incentives to run after unfavorable accounting signals. If the information system is sufficiently conservative, the bank prefers to offer a rent instead of incurring losses from premature runs.

*Previous versions have been presented at the EAA congress 2016, the Accounting Panel of the German Economic Association 2016, and the ÖFG Workshop 2015 in Graz, Austria. We would like the participants for helpful comments, in particular Tim Baldenius, Anna Boisits, Thomas Hemmer, Christian Laux, Jack Stecher, and Alfred Wagenhofer. This work was started while we were guests at the Jesse H. Jones Graduate School of Business at Rice University.
1 Introduction

The years 2018 and 2019 mark an important change regarding the accounting for loan contracts. The backward-looking "incurred loss" impairment model is replaced by the forward-looking "expected loss" model. In the language of the recently developed literature on Bayesian Persuasion, this can be regarded as a change from an uninformative information system to a conservative system where forward looking good news is concealed, but forward looking bad news is disclosed.

The modern discussion about the role of conservatism starts with Watts & Zimmermann (1986) and Watts (2003) who suggest that accounting conservatism enhances the efficiency of debt contracts. Quite soon this suggestion has been challenged by models of Bayesian Persuasion (Göx and Wagenhofer 2009, 2010 and Gigler, Kanodia, Sapra, Venugopalan 2009). The main result of these models is that not conservative, but aggressive accounting supports the efficiency of debt contracts.\(^1\)

In this paper we ask under which circumstances conservatism can be good if banks finance their long-term loans through short-term deposits. Following usual conventions, we call an information system conservative, if favorable signal realizations identify economic success reliably, but unfavorable realizations do not allow to conclude reliably that there is failure. Vice versa, the system is aggressive, if unfavorable signal realizations identify economic failure with sufficient reliability, but favorable signals do not allow to conclude that there is success.

\(^1\)To be more precise, Göx and Wagenhofer 2009 show that an asset should be measured at some fixed book value, combined with a forward-looking impairment rule. While this appears like a conservative rule at first sight, their results rests on the fact that rational balance sheet readers over-value the asset at the impairment threshold.
We investigate the impact of conservative vs aggressive measurement on the bank’s deposit contract design. We start from the observation that banks have an incentive to write deposit contracts that leave no rents to the depositors. The disadvantages are strong incentives to run after unfavorable interim news. These runs are costly, because they must be financed by premature liquidation of loans in an illiquid market. With no rents for the depositors, the bank ultimately bears the illiquidity cost. The problem can be overcome by promising the depositors higher face values for the deposits that leave the depositors an expected rent. We show that there exist situations with a sufficiently conservative information system where the bank prefers the latter sort of deposit contract, but favors the contract with more frequent runs when the information system is aggressive.

Consistent with the features of the 2008 crisis, we assume that the bank’s loans finance long-term assets in the real economy, and that the ability of the borrowers to repay the loans depends directly on the performance of those real assets. Thus, a negative movement of the real asset values affects performance of the loans which indirectly affects the bank’s ability to repay the deposits. Different from Morris and Shin (2000) and Goldstein and Pauzner (2005) this excludes the possibility of fundamental-based runs. The only reason for a run is panic, i.e. the depositors receive signals on the future of the bank’s loans that mistakenly characterize a performing loan as not performing. Under these circumstances, an aggressive information system that is free from biasing good outcomes as bad, avoids the cost from running.\(^2\) Of course, the reverse conclusion holds too. The expected cost from pre-mature runs cannot be avoided, if the information system sometimes characterizes

\(^2\)This is a corollary to Gigler, Kanodia, Sapra, and Venogopalan 2009 with the modification that costs from undue optimism are absent in our model.
performing loans as not performing. This is a typical feature of conservative information systems.

This reverse case is of particular interest to us if the model has a second equilibrium. Instead of running after an unfavorable signal, depositors might also consider to give up and stay because the cost of liquidation exceeds the benefit. If this holds in equilibrium, efficiency is restored for a different reason.

We show that this second equilibrium will be played if the face value of the deposits is sufficiently high. In this case, the social cost from inefficient asset liquidation is avoided. However, there is a caveat from the bank’s point of view. To establish the second equilibrium, the bank must leave rents to the depositors as the bank has to offer a sufficiently high face value of debt.

We conclude that when writing the deposit contract in the first place, the bank faces a trade-off: Leave no rent to the depositors, but suffer from costs of inefficient pre-mature runs, or incur the cost of the rent. The more aggressive the information system, the lower are the costs from running. Then, the bank prefers the equilibrium with inefficient runs but without depositors’ rents. However, if the system is more conservative, the bank prefers the equilibrium with depositors’ rents but reduced running.

**Related literature**

We build on the literature on Bayesian Persuasion (Kamenica and Gentzkow 2011; Taneva 2015; Bergemann and Morris 2016). Applied to accounting, Göx and Wagenhofer (2009, 2010) show that aggressive measurement of a pledged asset is beneficial for debt contracting in order to correct incentives
for under-investment in the presence of moral hazard. In the same vein, Gigler, Kanodia, Sapra, Venugopalan (2009, GKS) show that aggressive measurement of the interim success of the investment project reduces losses from inefficient runs and, in turn, under-investment in the first place. Our model borrows several element from GKS, but introduces multiple depositors and excludes fundamental based runs, because there is no fixed value of some alternative real asset.

Bank runs have been analyzed early by Diamond and Dybvig (1983). Like in their model, the run-vs-stay decisions of the shareholders may be strategic complements and, hence, there are multiple equilibria. Most recent models use the Global Games framework (Carlsson & van Damme 1993; Morris & Shin 2002 / 2004) as equilibrium refinement method in order to come to point predictions.\(^3\) Like Ebert, Kadane, Simons, and Stecher (2016) we use the dominance criterion by Harsanyi and Selten (1988) to select the equilibrium.\(^4\) Gao and Jiang (2018) show that under the threat of a run, managers with poor pre-manipulation earnings tend to manipulate the earnings report upward to the withdrawal threshold level of earnings, thus pooling themselves with better types. Similar to the first equilibrium in our model, such locally aggressive accounting reduces runs.

Our paper complements the vast literature on transparency of the banking system, as reviewed in Goldstein and Sapra (2013). Similar to our modeling this literature focusses on questions around the disclosure of bad information. Goldstein and Leitner (2015) show that a regulator should not disclose stress test results if the overall state of the industry is perceived as strong, while

\(^{3}\)This method is widely applied, see Goldstein & Pauzner (2005) or Gao & Jiang (2018).

\(^{4}\)This criterion assumes that the creditors aim at minimizing the negative effects of being wrong about equilibrium selection.
partial disclosure is optimal otherwise. Bouvard et al (2015) derive parallel results, supplemented by an analysis of the regulator’s incentives to publish the stress test results. These results are consistent to our starting point, namely, that aggressive information design reduces inefficient runs.\footnote{Extending these findings to the generation of information in the first place, Prescott (2008) makes the point that requiring bank supervisors to disclose their private information to the public makes it difficult for supervisors to collect information.}

2 The basic model

A bank finances a long term project with short term loans. Consider a three stage game with a time line as in Figure 1. Start of the project is at date 0, and the project has uncertain payoffs at date 2 in the future. There are two unknown states, good $g$ and bad $b$. Let the project’s rate of return at date 2 in the good state $g$ be $x_g = X > 1$ and $x_b = 0$ in the bad state $b$. The ex ante probability for the good state is $p$.

\begin{figure}[h]
\centering
\begin{tabular}{l l l}
\hline
\textbf{date 0} & \textbf{date 1} & \textbf{date 2} \\
\hline
Deposit contracts: & Signal $y$ reported & If bank didn’t crash \\
Cash inflow: $2I$ & & at date 1: \\
Nominal debt $2D$ & Depositors decide & Return on remaining \\
Bank invests $2I$ & whether to withdraw & capital realized \\
& if so, Bank liquidates & \\
& loans & \\
\hline
\end{tabular}
\caption{Time line}
\end{figure}

Two risk neutral symmetric depositors each lend an amount $I$ to the bank.
The bank invests $2I$ to finance its project. Let $D \geq I$ denote the chosen face value of the deposits. As the game approaches date 2, each depositor receives $D$ if the project is successful. To avoid tedious case distinctions, we assume that the bank has no equity or other businesses for funding. For simplicity the discount factor is 1. Thus, the ex ante expected net present value of the project is $2I(pX - 1)$ and the ex ante expected profit of the bank is $2p(XI - D)$.

At date 1, there is a public signal $y$ that is correlated with the date-2 rate of return $x$. We normalize $y$ to $y \in [0, 1]$. Higher values of $y$ are more optimistic news. Given the two states of the world, the signal density is denoted by $f(y|g)$ in the good state where the project is successful in the end. Moreover, the density is $f(y|b)$ in the bad state. To simplify, we assume that $f(y|g)$ is monotonically non-decreasing in $y$ while $f(y|b)$ is monotonically non-increasing in $y$.

Given a signal $y$, all parties update the probability for state $g$ to

$$q(y) = \frac{pf(y|g)}{pf(y|g) + (1 - p)f(y|b)}.$$  \hspace{1cm} (1)

We restrict the parameters such that $f(0 | g) = 0$ and $f(1 | b) = 0$. This excludes corner solutions regarding $q(y)$.

To keep the model parsimonious, we assume that depositors have the right to terminate the contract prematurely and claim back $I$ at date 1. Other papers endogenize this assumption by introducing income shocks (e.g. Diamond & Dybvig 1983, Dang, Gorton, Holmström, & Ordoñez 2017), or assuming that premature exit gives access to a safe asset (e.g. Morris & Shin 2000, Gao & Jiang 2018). In both cases, the contract will include covenants. Our setting is kept more simple. We exogenously do not restrict premature terminations.
As the bank has invested the full amount $2I$ at date 0, it has to finance early withdrawals by selling (parts of the) loans. We assume that the market for loans is illiquid. The market discounts the expected value of the loans by a factor $\lambda < 1$. Therefore, if the un-discounted value of a loan unit is $E[\tilde{x}|y] = q(y)X$, the liquidation value at $t = 1$ is $\lambda q(y)X < E[\tilde{x} | y]$. Hence, any withdrawal unit erodes the bank’s capital by $1/\lambda > 1$. Note that it is never efficient to liquidate the banks assets at date 1 in such a setting. In the language of the literature, any withdrawal is “panic based” in our model.

Given these assumptions, the bank is endangered at two dates. There is bankruptcy if the project fails at date 2. In addition, withdrawals at date 1 may lead to bankruptcy. Depending on $y$, the bank’s capital may be insufficient to finance the withdrawals. At date 1, bankruptcy may be triggered under three scenarios

1. The signal realization $y$ is sufficiently unfavorable such that the bank cannot pay out two required withdrawals, $2I$.
2. The signal realization $y$ is sufficiently unfavorable such that the bank cannot even pay out a single withdrawal amount, $I$.
3. After only one withdrawal, the bank can pay out $I$, but the remaining capital is too low to pay out $D$ to the remaining depositor at date 2.

We assume that if the bank crashes after two withdrawals (scenario 1), the liquidation proceeds $2\lambda q(y)XI$ are symmetrically shared between the depositors. In the case of bankruptcy after a single withdrawal (scenarios 2 & 3), the liquidation procedure is more complex and costly. In this case, coordinating with the remaining depositor causes an additional legal cost such that
total liquidation proceeds are \( \delta 2 \lambda q(y) XI \) with \( \delta \in (0, 1) \). These proceeds are again symmetrically shared between depositors.

Before we solve the model, consider the properties of the information system more closely. All properties of this system are technically embedded in the density functions \( f(y|g) \) and \( f(y|b) \). Precision in the bad state is depicted by the probability mass attached to the low signal realizations in this case. Improving the precision in the bad state means shifting more probability mass to low signal realizations and reducing the probability of false high signals. Similarly, precision in the good state is characterized by the probability mass of high signal realizations in the good state. Reducing the precision in the good state means shifting probability mass away from the high towards low signal realization.

Conservative differ from aggressive information systems in their precision in the good versus the bad state. We call an information system with equal precision in the good and the bad state neutral. Starting from a neutral information system, the system gets more conservative if the precision in the good state decreases, or the precision in the bad state increases. Likewise, the system gets more aggressive if the precision in the bad state decreases or the precision in the good state increases.

To illustrate this, consider two examples that we will use later. First, consider stepwise density functions with \( \gamma, \omega \in [0, 1] \).

\[
\begin{align*}
f(y|g) &= \begin{cases} 
1 - \gamma & y \in Y_1 = [0, 1/2) \\
\gamma + 1 & y \in Y_2 = [1/2, 1]
\end{cases} \\
f(y|b) &= \begin{cases} 
\omega + 1 & y \in Y_1 = [0, 1/2) \\
1 - \omega & y \in Y_2 = [1/2, 1]
\end{cases}
\end{align*}
\]
This information system is equivalent to a binary signal. The signal exhibits bad news if \( y \) is between 0 and \( 1/2 \) and good news if \( y \) is between \( 1/2 \) and 1. In the special case where \( \gamma = \omega = 1 \), the signal is *precise*. If \( \omega = \gamma = 0 \), the system is completely *uninformative*. More interestingly, the information system gets more conservative if \( \gamma \) decreases or \( \omega \) increases. Vice versa, the information system becomes more aggressive if the value of \( \omega \) decreases or \( \gamma \) increases.

Second, consider the following class of linear density functions \( f(y|g) \) and \( f(y|b) \). Let

\[
\begin{align*}
f(y|g) &= 1 - \gamma + 2\gamma y, \quad \gamma \in [0; 1] \\
f(y|b) &= 1 + \omega - 2\omega y, \quad \omega \in [0; 1].
\end{align*}
\]

(3)

Again, \( \gamma \) is a measure of precision in the good state. For \( \gamma = 1 \), the signal is most precise, even though not perfectly precise. For \( \gamma = 0 \), precision is lowest. Similarly, if the state of the world is bad, \( \omega \) measures precision in the bad state. Again, the information system is more conservative if \( \gamma \) lower or \( \omega \) is higher and vice versa.

### 3 Withdrawal equilibria

Depositors’ behavior at date 1 depends on the signal they receive. We restrict attention to symmetric equilibria only. Three possible cases emerge.\(^6\)

\(^6\)The formal analysis is relegated to the proofs and can be found in the Appendix.
**Case 1.** Suppose the signal is *very favorable*, such that the emerging price from liquidating loans is sufficient cover withdrawals of two loans, i.e. \(2\lambda q(y)XI > 2I\). This yields
\[
q(y) > \frac{1}{\lambda X}.
\]

Given such a favorable signal, each depositor knows that a withdrawal yields \(I\) for sure. On the other hand, a decision to stay yields an expected payoff of \(q(y)D\). Thus, each depositor decides to withdraw if
\[
D < \frac{I}{q(y)}
\]
and stay otherwise.

**Case 2.** Consider an *intermediate signal* such that the bank survives a single withdrawal, but not two withdrawals. After one withdrawal, the bank’s remaining capital is \(2I - \frac{I}{\lambda q(y)X}\). This won’t be sufficient to pay out \(D\) if
\[
D > X \cdot \left(2I - \frac{I}{\lambda q(y)X}\right) \quad \text{or} \quad q(y) > \hat{q}(D) = \frac{I}{\lambda(2XI - D)}
\]
where \(\hat{q}(D)\) is the critical posterior probability of success such that the bank must declare bankruptcy after one withdrawal. Hence, the condition for Case 2 is
\[
q(y) \in \left[\hat{q}(D), \frac{1}{\lambda X}\right].
\]

Now consider the depositors’ incentives in this case. Given that the other depositor does not withdraw, the best response is to stay if \(q(y)D\) exceeds \(I\). Thus, we get as equilibrium
\[
\text{(stay-stay) if and only if } D \geq I/q(y).
\]
However, the condition for a withdrawal equilibrium changes. Given that the other depositor withdraws, withdrawing yields half of the total liquidation proceeds, i.e. $\lambda XI q(y)$, while not withdrawing yields an expected payoff of $q(y)D$. Thus, we get

$$(\text{run, run}) \text{ if and only if } D < \lambda XI.$$ 

Finally, $D$ might be between $I/q(y)$ and $\lambda XI$. Then, the best reply to withdrawing it to stay and vice versa. This results in a symmetric mixed strategy equilibrium where the withdrawal probability $r(y)$ is

$$r(y) = \frac{I - q(y)D}{I - \lambda XI q(y)}.$$ 

**Case 3.** Next, consider an *unfavorable signal* where

$q(y) < \hat{q}(D).$

Even a single withdrawal erodes the capital base so much that repaying the remaining depositor becomes impossible and the bank has to declare bankruptcy at date 1. This case will drive our results. If liquidation proceeds become really low, no simple stay-run equilibrium arises. Instead staying becomes attractive if the face value of the deposits $D$ is sufficiently high. However, there is a coordination problem: If one depositor withdraws, the second depositor’s best reply is to withdraw too. Given that the other depositor stays, staying is a best response if the face value $D$ is sufficiently high, i.e.

$$D \geq (1 - \delta) \lambda XI$$

Thus we have multiple equilibria here.

Multiple equilibria call for a prediction which equilibrium will be played. Most of the literature uses global games as equilibrium refinement (Carlsson...
& van Damme 1993; Morris & Shin 2002 / 2004). Like Ebert et al (2016) we use to Harsanyi’s and Selten’s (1988) risk dominance criterion. According to this criterion, the (stay, stay) equilibrium will be chosen if and only if the difference in payoffs from individually deviating from the stay-stay equilibrium exceeds the difference from deviating from the run-run equilibrium. Formally,

$$q(y)D - (1 - \delta)\lambda q(y)XI > \lambda q(y)XI - (1 - \delta)\lambda q(y)XI$$
or

$$D > \lambda XI.$$  

Summarizing these three cases yields the following Proposition.

**Proposition 1** Applying the risk dominance criterion yields the following equilibrium patterns at the withdrawal stage

1. For a low face value of deposits, $D < \lambda XI$, depositors stay if $q(y) \geq I/D$ and withdraw otherwise.
2. For $D \geq \lambda XI$ they stay if $q(y) \geq I/D$, withdraw with probability $r(y)$ if $q(y) \in [\hat{q}(D), I/D)$, and stay if $q(y) < \hat{q}(D)$.

4 Withdrawal Equilibrium Patterns

As summarized in Proposition 1, the equilibrium at the withdrawal stage depends on the signal realization, but also the face value $D$. The key takeaway from the Proposition is that by choosing the face value $D$, the bank selects between two entirely different patterns at the withdrawal stage. If $D$ is low, the equilibrium is to withdraw if the signal realization is bad, and to stay if it is good. A similar pattern arises in the majority of the bank-run and debt contracting literature. We call this *Equilibrium Pattern A*. 

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However, for $D$ sufficiently high, the above bang-bang equilibrium does not exist. In particular, for $D \geq \lambda XI$, depositors will no longer withdraw for the very bad signal realizations. The intuition is that liquidation proceeds erode so much that it is to gamble against the bad odds and stay. We refer to this as *Equilibrium Pattern B*.

**Equilibrium Pattern A**

Under Equilibrium Pattern A, depositors stay for all signals above the withdrawal boundary where, $q(y) > q^D = q(y^D) = I/D$, and withdraw otherwise. Moreover, if depositors withdraw, the bank survives the withdrawals if and only if $y$ exceeds some critical value $y^*$ with $q(y^*) = \frac{1}{\lambda X}$. Notice that $y^* < y^D$.

The bank’s profit maximizing problem is

$$\max_D p \left(1 - F(y^D | g)\right) 2(XI - D) + \int_{y^*}^{y^D} 2I \left(1 - \frac{1}{\lambda Xq(y)}\right) xpf(y | g)dy$$

s.t. $D < \lambda XI$ (Condition for Equilibrium Pattern A)

$D \geq D^{\text{part}}$ (Depositors’ participation constraint).

Before we start, recall from Proposition 1 that an Equilibrium Pattern A only exists if the face value $D$ is sufficiently low. Thus, a necessary condition is that $D^{\text{part}}$ must be lower than $\lambda XI$. This is equivalent to $\lambda pX > 1$. Hence, Equilibrium Pattern A only exists if the ex ante NPV is sufficiently high such that each invested unit is covered ex ante by the fictitious immediate liquidation amount.

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7This is trivial to see in the binary example below. We prove this claim for the general case in the Appendix.
We start deriving the solution by solving the bank’s unconstrained problem first. The unconstrained tradeoff is that reducing $D$ lets the bank’s profit in the stay-area increase, but also lets the withdrawal area grow. Denote the solution to this unconstrained tradeoff by $D^*$ where

$$D^* = I \cdot \frac{1 - \lambda}{\lambda} \frac{f(y^D | g)}{1 - F(y^D | g)} \frac{\partial y^D}{\partial q}.$$

If $D^*$ satisfies the participation constraint and $D^* < \lambda XI$, the solution has been found. Notice that $D^*$ is decreasing in $\lambda$. For $\lambda \to 1$ it assumes a corner solution and equals $I$. Obviously, $I$ is lower than $D_{part}$ and therefore, the participation constraint is binding for sufficiently high liquidity. Moreover, we cannot rule out that there exist cases with sufficiently low $\lambda$ such that the participation constraint is slack. While we cannot characterize those cases in the general model, we identify them in numerical examples below.

**Lemma 1** Equilibrium Pattern A is feasible only if $\lambda pX > 1$. For liquid projects, ($\lambda$ high), the entire expected surplus goes to the bank and the depositors’ participation constraint is binding.

**Equilibrium Pattern B**

We now assume that the bank is restricted to suggest a deposit contract such that Equilibrium Pattern B emerges at date 1. To keep the following comparable to what happens under Equilibrium Pattern A, we assume the identical parameter set. In particular, we assume that $\lambda pX > 1$ as Equilibrium Pattern A doesn’t exist otherwise. Recall that the following equilibrium pattern emerges if $D \geq \lambda XI$.

- Depositors stay for unfavorable realizations with $q(y) < \hat{q}(D)$. 

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• Depositors stay for the favorable signal realizations with $q(y) \geq q^D = I/D$.

• Depositors withdraw with probability $r(y)$ for all intermediate signals.

This yields the following optimization problem. The bank maximizes

$$
\Pi^B = \int_0^{\hat{y}} 2p(XI - D)f(y|g)dy + \int_{y_D}^{\hat{y}} 2p(XI - D)f(y|g)dy \\
+ \int_{\hat{y}}^{y_D} 2pf(y|g) \left[ r(1 - r) \left( x \left( 2I - \frac{I}{\lambda X q(y)} \right) - D \right) + (1 - r)^2 (XI - D) \right] dy
$$

\text{s.t.} \quad D \geq \lambda XI \quad (Equilibrium type condition) \\
D \geq D^{part} \quad (Depositors' Participation constraint).

Different to Equilibrium Pattern A, Pattern B also exists for $\lambda X p < 1$. In particular, extremely low liquidity $\lambda < \frac{1}{2pX - 1}$ implies no runs at all and the First Best becomes possible. High liquidity has the potential to destroy the equilibrium pattern. For $\lambda \to 1$, $D$ converges to $XI$, and the bank’s expected profit vanishes.

Now, a marginal change of $D$ has three effects.

$$
\frac{\partial \Pi^B}{\partial D} = -2p + 2p \left( \frac{1 - \lambda}{\lambda} \right) \int_{\hat{y}}^{y_D} 2Dq(y) - I \frac{I}{I - \lambda X I q(y)} f(y|g)dy \\
+ 2pD \left( \frac{1 - \lambda}{\lambda} \right) r(y) \frac{\partial \hat{q}}{\partial D} f(\hat{y}|g) = 0
$$

According to (I), increasing the face value of the deposits has a direct, negative effect on expected bank profits. However, there are two positive indirect effects. According to expression (II) the increase of $D$ reduces absolute
amount of the withdrawal probability \( r(y) \). Moreover, according to (III), an increase in \( D \) lets the middle region of mixed strategy withdrawals shrink. To illustrate the two indirect effects, start with \( D = \lambda XI \). Inserting this into \( r(y) \), we obtain that \( r(y) = 1 \) for all intermediate signal realizations. Moreover, the distance between \( \hat{q}(D) \) and \( q^D \) is maximized. Marginally increasing \( D \) reduces both the depositors’ withdrawal probabilities and the length of the withdrawal interval.

Without additional model structure there are no straightforward results to the above tradeoff. Denoting the solution to the unconstrained problem by \( D^{**} \), it is impossible to say whether \( D^{**} \) exceeds or does not exceed the minimum amount for Equilibrium Pattern B, given by \( \lambda XI \). In the examples below we identify a set of cases where the \( D \) assumes the corner solution \( \lambda XI \).

The Bank’s Choice

The most interesting question is which equilibrium pattern the bank prefers, given that both patters coexists. Thus, we restrict ourself to \( \lambda p X > 1 \). Unfortunately, the general formulation of our model does not allow for explicit solutions. Therefore, we use this section to develop the basic intuition for the results and confirm it by two examples further below.

Equilibrium patterns A and B differ with respect to two properties: First, the face value of deposits \( D \) is greater under B than under A. In particular, for \( \lambda p X > 1 \) the depositors’ participation constraint is not binding under pattern B. Hence, the bank must leave an expected rent to the depositors. Second, the ex ante probability of premature runs and deadweight losses is significantly increases, once the bank reduces \( D \) from \( D = \lambda XI \) to some
amount slightly lower. Under Equilibrium Pattern A the bank fully bears this increase if the depositors’ participation constraint is binding.

Notice that the inefficiency from running is caused by imprecise measurement that triggers false alarms. It follows that if the precision in the good state is sufficiently high, the bank always prefers Equilibrium Pattern A over B. Put otherwise, once measurement precision in the good state is low, the bank might consider raising $D$ and switching to a deposit contract that induces Equilibrium Pattern B. From a social perspective, this is desirable because at the margin $D = \lambda XI$, Equilibrium Pattern B is more efficient than A.

Ceteris paribus, information systems with a low precision in the good state are conservative. Moreover, conservative systems are often also associated with high precision in the bad state. However, as there is no cost or benefit from wrong measurement in the bad state in our model, measurement precision in the bad state has no first order effects on our key result.

5 Two Examples

Binary case

It is hard to derive general solutions to our model. Hence, it is helpful to develop illustrative examples. To this end we first work with the binary signal case, as introduced in (2) above. Updating leads to one of two possible posterior probabilities of success, $q_1$ or $q_2$ where $q_1 = \frac{p(1-\gamma)}{p(1-\gamma)+(1-p)(1+\omega)}$ denotes unfavorable prospects after $y \in [0,1/2) = Y_1$ and $q_2 = \frac{p(1+\gamma)}{p(1+\gamma)+(1-p)(1-\omega)}$ denotes favorable prospects after $y \in [1/2,1] = Y_2$. To keep our illustrative
example as simple as possible, we select the parameters such that \( q_1 \) is "very" low, i.e. \( q_1 \leq \hat{q}(\lambda XI) \) and that \( q_2 \) is sufficiently favorable such that \( q_2 > I/D \).

If the bank sets \( D \geq \lambda XI \), the First Best is attained. However, if the bank offers \( D < \lambda XI \), depositors run after the bad signal and stay after the good signal. As runs exhibit social losses, the First Best is missed.

The remaining question is how to give the bank incentives to offer \( D \geq \lambda XI \) voluntarily. We proceed in two steps: we first solve the bank’s maximization problems conditional on \( D < \lambda XI \) (Equilibrium Pattern A) and \( D \geq \lambda XI \) (Equilibrium Pattern B). In a second step we compare the bank’s profits from the two problems and derive a condition under which the bank prefers B over A.

Restate the bank’s maximization problem. If the bank aims to establish Equilibrium Pattern A, the problem reads

\[
\max_D \quad 2p \cdot Pr(y \in Y_2 \mid g)(XI - D)
\]

such that

\[
(1A) \quad p \cdot Pr(y \in Y_2 \mid g)D + (p \cdot Pr(y \in Y_1 \mid g) + (1 - p) \cdot Pr(y \in Y_1 \mid b))\lambda q_1 XI \geq I
\]

\[
(2A) \quad q_2 D \geq I
\]

\[
(3A) \quad q_1 D < \lambda q_1 XI
\]

\[
(4A) \quad D < \lambda XI
\]

Constraint (1A) is the participation constraint. Constraint (2A) ensures that depositors stay after favorable news (i.e. \( y \geq 1/2 \)). Constraint (3A) ensures that depositors run after unfavorable news (i.e. \( y < 1/2 \)). Finally, constraint (4A) ensures that the resulting equilibrium pattern is of type A. Quick substitutions show that (4A) is equivalent to \( \lambda pX > 1 \) if the participation constraint (1A) is binding. We obtain the following result
Proposition 2 Suppose that the ex ante expected NPV is high, \( \lambda pX \geq 1 \). Moreover, suppose that the information system satisfies

\[
\omega \geq \frac{p}{1-p} (\lambda X - 1)(1 - \gamma) - 1.
\]

Then, the bank can implement a (stay-run) equilibrium by offering

\[
D_{\text{part}} = \frac{2 - \lambda p X (1 - \gamma)}{p(1 + \gamma)} \cdot I.
\]

In this contract, the expected bank profit is

\[
\underbrace{2(pXI - I)}_{\text{expected NPV}} - \underbrace{2pXI(1 - \lambda)}_{\text{expected loss}} \cdot \frac{1 - \gamma}{2}.
\]

Corollary 1 Suppose that \( \lambda pX > 1 \). If \( \gamma \to 1 \), the bank attains the First Best expected profit.

Under Equilibrium Pattern A, the bank’s expected profit is reduced because of the expected deadweight loss through premature runs after false alarm. Notice that this loss goes to zero if the information system maps the good state into a favorable signal realization \( y_2 \) for sure. The latter result is in line with previous research in debt contracting. Avoiding false alarms is more beneficial than avoiding undue optimism (e.g. Gigler et al. 2009).

Now suppose that under the assumptions of the previous Proposition \( \gamma \) is bounded away from its maximum value 1, such that false alarms occur with sufficiently high probability. The question is under which circumstances the bank prefers to offer \( D \geq \lambda XI \) instead of \( D_{\text{Part}} \) such that Equilibrium Pattern B emerges. From the bank’s perspective, the problem with Equilibrium Pattern B is that \( D \geq \lambda XI \) leads to a non-binding participation constraint.
While the total surplus attains its First Best level, part of it must be left as a rent to the depositors.

Conditional on implementing Equilibrium Pattern B, the bank maximizes

\[2p(XI - D)\]

such that

\[(1B) \quad D \geq \frac{I}{p} \quad \text{ex ante participation constraint}\]

\[(2B) \quad D \geq \frac{I}{q_2} \quad \text{rollover constraint after } y_2\]

\[(3B) \quad D \geq \lambda XI \quad \text{rollover constraint after } y_1\]

\[(4B) \quad q_1 \leq \hat{q}(D) \quad \text{or } D \geq 2XI - \frac{I}{q_1\lambda} \quad \text{depositors stay after } y_1.\]

Given that we keep \(q_1\) and \(q_2\) identical to Proposition 2 and just introduce the additional assumption that \(D \geq \lambda XI\), it is obvious that (2B) must be slack. It remains to state a condition under which (3B) is binding and (4B) slack. This holds if

\[\lambda XI \geq 2XI - \frac{I}{q_1\lambda},\]

which can be rewritten as

\[\omega \geq \frac{p}{1-p}[(2\lambda - \lambda^2)X - 1](1 - \gamma) - 1.\]

We conclude

**Proposition 3** Suppose the expected NPV is high, \(\lambda pX \geq 1\), and the information system is sufficiently aggressive in the sense that

\[\omega \geq \frac{p}{1-p}[(2\lambda - \lambda^2)X - 1](1 - \gamma) - 1.\]
Then, the bank can implement a no-run equilibrium by offering $D = \lambda XI$. In this case, the bank’s expected profit is given by

$$2pXI(1 - \lambda).$$

Before we proceed, we should repeat that Propositions 2 and 3 do not provide a full characterization of the problem. Rather, they identify one particular parameter set that allows to compare the two equilibrium patterns.

Next, we restrict attention to the intersection of both feasible sets as defined by (4) and (5). We ask which of the two solutions the bank prefers. The comparison is straightforward for $\gamma = 1$, because then, the expected costs from running are nil under Equilibrium Pattern A and the First Best is achieved. Hence, a necessary condition that the bank prefers Equilibrium Pattern B over A is that $\gamma$ is sufficiently low and the associated expected costs from running are sufficiently high. Compare the expected profits under A and B,

$$2pXI(1 - \lambda) \geq 2(pXI - I) - 2pXI(1 - \lambda) \cdot \frac{1 - \gamma}{2},$$

to obtain:

**Proposition 4** Suppose that both a (stay-run) and a (stay-stay) contract are feasible. The bank prefers a (stay-stay) contract to a (stay-run) contract if and only if (**new rechnen**)

$$\gamma \leq \hat{\gamma} = \frac{2 - pX[3\lambda - 1]}{pX(1 - \lambda)}.$$

Proposition 4 is the main result of this example. Suppose both equilibrium patterns are possible for a particular $\gamma$. In order to commit the bank to
choose a more efficient Equilibrium Pattern B, the information system must be sufficiently imprecise when mapping the good state into signals. This property typically holds for conservative information systems.

**Continuous linear case**

In this section, we illustrate the results by using the linear density functions defined in (3) above. To repeat, in the good state the density for \( y \) is 
\[
f(y|g) = 1 - \gamma + 2\gamma y, \quad \gamma \in [0; 1]
\]
and in the bad state it is 
\[
f(y|b) = 1 + \omega - 2\omega y \quad \text{where} \quad \omega \in [0; 1].
\]
\( \gamma \) is a measure of precision in the good state and \( \omega \) measures precision in the bad state. For \( \gamma = 1 \) (\( \omega = 1 \)), the signal assumes maximum possible precision in the good (bad) state while for \( \gamma = 0 \) (\( \omega = 0 \)), precision in the good (bad) state is lowest.

Notice that different to the binary example above, the case of \( \gamma = \omega = 1 \) refers to the case with maximum attainable precision, but still there is imprecision by construction.

We derive the solution to the problem numerically. As before, we first solve the problems A and B conditional on \( D < \lambda XI \) and \( D \geq \lambda XI \) and then compare expected bank profits. We then pointwise derive the threshold where the bank chooses Equilibrium Pattern B instead of A.

Figure 2 shows how the bank’s expected profit function depends on the level of the face value of deposits, \( D \). In the two examples the feasible set starts with \( D^{\text{part}} \) as determined through the participation constraint. In the two examples, expected profit is decreasing in \( D \) at \( D^{\text{part}} \).

\[8\]

Notice that this need not be the case in general. In particular, for low values of \( \gamma \) and \( \omega \) we obtain results where the participation constraint is not binding.

\[22\]
equilibrium pattern at the withdrawal stage switches. For $D \geq \lambda XI$ there are no withdrawals for low realizations of $y$. This leads to an upward jump of the expected profit function.

In the left figure, this upward jump is not sufficient to switch to Equilibrium Pattern B and the bank’s optimal level of $D$ is $D^{part}$. Consistent with the results of the binary example, we obtain this result for a high level of precision in the good state. If we reduce the precision $\gamma$, the result changes: in the right figure, expected profit assumes the global maximum at $D > \lambda XI$. Thus, for sufficient imprecision in the good state, the bank voluntarily chooses a high level of $D$ and the depositors gather a rent.

In the next step we generalize the result for different values of $\gamma$ and $\omega$ and obtain the results that are summarized in Figures 3 and 4. Figure 3 depicts two thresholds. The vertical line is the border at which the bank switches from Equilibrium Pattern A ($D < \lambda XI$) to Equilibrium Pattern B.
Figure 3: Illustration of model results for \( p = 0.5, X = 3, \lambda = 0.72, I = 1 \) (\( D \geq \lambda XI \)). As can be seen, this border is almost vertical, i.e. the bank’s tradeoff depends strongly on the precision in the good state, but less so on precision in the bad state. Different from the binary example the threshold is not exactly independent of the precision in the bad state because \( \omega \) enters the withdrawal probability \( r(y) \).\(^9\) As before, the bank chooses Equilibrium Pattern A if the precision in the good state is high, and B if it is low.

The figure also contains a diagonal line. Right to this line, the depositors’ participation constraint is binding under Equilibrium Pattern A. For the parameter values in Figure 3 this is not relevant, because for those parameter values where the participation constraint is not binding the bank chooses

\(^9\)This complication has been avoided through the construction of the binary example.
Figure 4: Illustration of model results for $p = 0.5, X = 3, \lambda = 0.73, I = 1$
chooses Equilibrium Pattern B anyway. However, the threshold becomes important if we slightly change the parameters. Figure 4 shows the latter case. In this case, the bank prefers to leave the participation constraint slack for low precisions in the bad state. Despite of leaving a rent to the depositors under Equilibrium Pattern A, it still prefers A over B, if the precision in the good state is sufficiently high. However, the threshold between B and A is tilted to the right in the intersection area. The reason is that now Equilibrium Pattern A has two costs: costs of excessive runs, but also a cost of leaving a rent to the depositors. The rent that must be left to the depositor is an additional cost for the bank under Equilibrium Pattern A. Hence, the bank is indifferent between A and B only if the precision in the good state $\gamma$ is higher.

6 Conclusion

Despite of forceful arguments pro conservatism in debt contracts in the verbal literature (Watts 2003), models of Bayesian persuasion come to the opposite result: an aggressive information system is beneficial and a conservatism is bad (Goex & Wagenhofer, 2009; GKSV 2009; Kamenica & Gretzkow 2011). We develop conditions such that a conservative information system is good. To this end, we develop a model where multiple depositors may withdraw their money from a bank. In equilibrium a sufficiently high face value of the deposits triggers rollovers of deposits even after bad information. Offering a contract with this face value becomes beneficial to the bank if the expected cost of premature withdrawals becomes sufficiently large. The latter happens if the information system maps success into bad signals with sufficiently high probability — a feature that is typically associated with conservatism.
Our result is analogous to the efficiency wage theory. Similar to the necessity to keep important workforces in a company by offering them wages above the reservation value, there is an advantage to offer debtholders contractual claims that include rents. We conclude that a low cost of debt in banks is not necessarily good. Rather, there is a tradeoff between higher cost of debt in good times and lower vulnerability in a crisis. This should be carefully taken into account in the financial analysis of banks.
7 Appendix ... still in its infancy!

7.1 Withdrawal behavior

A depositor’s payoffs if he withdraws is\(^\text{10}\)

\[
W(0; y) = \begin{cases} 
  I & q(y) \geq \frac{1}{2X} \\
  (1 - \delta)\lambda X q(y) I & q(y) < \frac{1}{2X}
\end{cases}
\]

\[
W(1; y) = \begin{cases} 
  I & q(y) \geq \frac{1}{X} \\
  \lambda X q(y) I & q(y) < \frac{1}{X}
\end{cases}
\]

where \(W(0; y)\) denotes the payoff if the other depositor does not withdraw, while \(W(1; y)\) stands for the case where both withdraw. Similarly, (expected) profits from leaving the money in the bank are

\[
V(0; y) = q(y)D
\]

\[
V(1; y) = \begin{cases} 
  q(y)D & q(y) \geq \hat{q}(D) \\
  (1 - \delta)\lambda X q(y) I & q(y) < \hat{q}(D).
\end{cases}
\]

Here, \(\hat{q}(D)\) denotes the bankruptcy boundary: Given the other depositor withdraws, the bank has to declare bankruptcy at date 1 if the remaining capital is insufficient to generate sufficient funds even in the success case

\[
\left(2I - \frac{I}{\lambda X q(y)} \right) X < D \quad \text{or} \quad q(y) < \hat{q}(D) = \frac{I}{\lambda (2XI - D)}.
\]

Start with a favorable signal such that bank can shoulder two withdrawals, \(q(y) \geq \frac{1}{X}\).

<table>
<thead>
<tr>
<th>payoffs</th>
<th>stay</th>
<th>withdraw</th>
</tr>
</thead>
<tbody>
<tr>
<td>stay</td>
<td>(q(y)D; q(y)D)</td>
<td>(q(y)D; I)</td>
</tr>
<tr>
<td>withdraw</td>
<td>(I; q(y)D)</td>
<td>(I; I)</td>
</tr>
</tbody>
</table>

\(^{10}\)Since \(q(y)\) is monotonically increasing in \(y\), we express all properties in \(q(y)\) instead of \(y\).
Thus, both stay if \( q(y)D \geq I \) or \( q(y) \geq q(y^D) = q_D = \frac{I}{D} \), and both leave otherwise.

Next, consider a really bad signal such that \( q(y) < \hat{q}(D) \) and the bank has to declare bankruptcy already after a single withdrawal. The payoff matrix now becomes

<table>
<thead>
<tr>
<th>payoffs</th>
<th>stay</th>
<th>withdraw</th>
</tr>
</thead>
<tbody>
<tr>
<td>stay</td>
<td>( q(y)D; q(y)D )</td>
<td>( (1 - \delta)\lambda XIq(y); (1 - \delta)\lambda XIq(y) )</td>
</tr>
<tr>
<td>withdraw</td>
<td>( (1 - \delta)\lambda XIq(y); (1 - \delta)\lambda XIq(y) )</td>
<td>( \lambda XIq(y); \lambda XIq(y) )</td>
</tr>
</tbody>
</table>

Both leave is an equilibrium for sure since \( (1 - \delta)\lambda XIq(y) < \lambda XIq(y) \).

Moreover, both depositors do not find it worthwhile to reclaim their money and gamble if \( q(y)D \geq (1 - \delta)\lambda XIq(y) \) or \( D \geq (1 - \delta)\lambda XIq(y) \). Thus, we may have multiple equilibria here. We need a prediction which equilibrium will be applied. Given the differences in the risk profile of the two equilibria, we apply Harsanyi’s and Selten’s risk dominance criterion. That is, the stay-stay equilibrium will be chosen if and only if \([q(y)D - (1 - \delta)\lambda XIq(y)] > [\lambda XIq(y) - (1 - \delta)\lambda XIq(y)] = \delta\lambda XIq(y)\). This yields \( q(y)D > \lambda XIq(y) \) or \( D > \lambda XI \). Thus, we get: both run if \( D < \lambda XI \) and both stay if \( D \geq \lambda XI \).

Finally, consider the case of intermediate signals where the bank does not have to declare bankruptcy after one withdrawal, but cannot pay out two withdrawals, \( q(y) \in [\hat{q}(D); \frac{1}{\lambda X}] \).

<table>
<thead>
<tr>
<th>payoffs</th>
<th>stay</th>
<th>withdraw</th>
</tr>
</thead>
<tbody>
<tr>
<td>stay</td>
<td>( q(y)D; q(y)D )</td>
<td>( q(y)D; I )</td>
</tr>
<tr>
<td>withdraw</td>
<td>( I; q(y)D )</td>
<td>( \lambda XIq(y); \lambda XIq(y) )</td>
</tr>
</tbody>
</table>

If \( q(y)D \geq I \) or \( D \geq \frac{I}{q(y)} \), both depositors stay. If \( \lambda XIq(y) > q(y)D \) or \( \lambda XI > D \), both leave. For \( \lambda XI \leq D < \frac{I}{q(y)} \) we have a symmetric mixed strategy equilibrium where the withdrawal probability \( r(y) \) is given by

\[
r(y) = \frac{I - q(y)D}{I - \lambda XIq(y)}.
\]

(6)

Note that we get \( r(y) = 1 \) if \( D = \lambda XI \) for all \( y \). Moreover, if \( D > \lambda XI \) we
have \( r(y) = 0 \) for \( q(y) = q^D = \frac{I}{D} \), \( \frac{\partial r(y)}{\partial q} < 0 \), but \( r(y) < 1 \) at \( q(y) = \hat{q}(D) \).

### 7.2 Equilibrium A

We start with the presumption that \( D < \lambda XI \). Thus, the withdrawal boundary is above the two-withdrawal survival threshold, \( q^D = \frac{I}{D} > \frac{1}{\lambda X} \). For all signals where \( y \geq y^D \) both depositors stay, and both leave for all \( y < y^D \).

The withdrawals lead to bankruptcy if \( q < \frac{1}{\lambda X} \) (denote \( y^* \) as \( q_{y^*} = \frac{1}{\lambda X} \)), but the bank survives if \( q \in \left[ \frac{1}{\lambda X}; q^D \right] \). The banks optimization problem therefore is

\[
\max_D p \left( 1 - F(y^D|g) \right) 2(XI - D) \\
+ \int_{y^*}^{y^D} 2I \left( 1 - \frac{1}{\lambda X q(y)} \right) xpf(y|g)dy \\
s.t. \quad D < \lambda XI \quad \text{(equilibrium type condition)} \\
\quad D \geq D_{\text{part}} \quad \text{(Depositors' Participation constraint)}
\]

where the depositors’ ex-ante participation constraint rewrites as

\[
\int_0^{y^*} \lambda XI q(y) \left[ p f(y|g) + (1 - p) f(y|b) \right] dy \\
\int_{y^*}^{y^D} I \left[ p f(y|g) + (1 - p) f(y|b) \right] dy + p \int_{y^D}^{1} D f(y|g) dy \geq I.
\]

This constraint reduces to

\[
D^A \geq D_{\text{part}} \equiv I \left[ 1 - \lambda X pF(y^*|g) \right] \\
\left( \frac{1}{p(1 - F(y^D|g))} - p \right) \frac{p F(y^D|g) - F(y^*|g)) + (1 - p)(F(y^D|b) - F(y^*|b))}{p(1 - F(y^D|g))}
\]
As we need to ensure $\lambda X I > D^A \geq D^\text{part}$, compare $D^\text{part}$ to $\lambda X I$.

\[
\lambda X I > I \frac{1 - \lambda X p F(y^*|g)}{p(1 - F(y^D|g))} - I \frac{p(F(y^D|g) - F(y^*|g)) + (1 - p)(F(y^D|b) - F(y^*|b))}{p(1 - F(y^D|g))}
\]

\[
\lambda X p > p + (1 - p) \frac{1 - F(y^D|b) + F(y^*|b)}{1 - F(y^D|g) + F(y^*|g)} = p + (1 - p)w
\]

Define the single crossing point $f(y^0|g) = f(y^0|b)$ as $y^0$. Note that $q(y^0) = p$. Suppose $\lambda X p < 1$ such that $p < \frac{1}{\lambda X}$ and $y^0 < y^* < y^D$. But since $f(y|g) > f(y|b)$ for $y > y^0$, we get $\frac{1 - F(y^D|b) + F(y^*|b)}{1 - F(y^D|g) + F(y^*|g)} = w > 1$. But for $w > 1$ we get $p + (1 - p)w > 1$. Thus, we get a contradiction $1 > \lambda X p > p + (1 - p)w > 1$.

In words, this equilibrium is impossible if $\lambda X p < 1$ as any $D^A$ that satisfies the participation constraint then would be too large to satisfy the equilibrium type constraint.

**Result 1** No equilibrium of type A exists if $\lambda X p < 1$.

Maximize the bank’s profits ignoring the participation constraint temporarily. From rewriting the profit as

\[
\Pi^A = 2p(X I - D)(1 - F(y^D|g)) + 2pXI \left( F(y^D|g) - F(y^*|g) \right) - \frac{2I}{\lambda} \left[ pF(y^D|g) - F(y^*|g) \right] + (1 - p) \left( F(y^D|b) - F(y^*|b) \right)
\]

the first-order condition

\[
0 = -2p \left( 1 - F(y^D|g) \right) - 2p(X I - D)f(y^D|g) \frac{\partial y^D}{\partial D}
\]

\[
+ 2pXI f(y^D|g) \frac{\partial y^D}{\partial D} - 2I \frac{1}{\lambda} \left[ pf(y^D|g) + (1 - p)f(y^D|b) \right] \frac{\partial y^D}{\partial D}
\]
yields
\[-2p \left[ 1 - F(y^D|g) \right] - 2pD \left( \frac{1}{\lambda} - 1 \right) f(y^D|g) \frac{\partial y^D}{\partial D} = 0 \quad (7)\]

and for \( \lambda \neq 1, \)
\[D^* = I \frac{1 - \lambda}{\lambda} \frac{f(y^D|g)}{1 - F(y^D|g)} \frac{\partial y^D}{\partial q}\]

It is not immediately clear whether the ex-ante participation constraint or the expression above determines \( D^A. \) Note that from equation (7) we get \( D^* = I \) for \( \lambda = 1, \) but this strictly violates the participation constraint. Thus, at least for large values of \( \lambda \) we get \( D^A = D^A_{Part}. \) For lower values of \( \lambda \) there exist cases where \( D^A = D^* > D^A_{Part}; \) existence proof by example. Without further assumptions, no simple rule can be provided as properties of the signal technologies play a complex role here plus non-linearity in \( \lambda. \)

### 7.3 Equilibrium B

Presuppose \( D > \lambda XI \) such that \( q^D = \frac{I}{D} < \frac{1}{AX}. \) We have three behavioral regions

- Stay for bad signals where \( q(y) < \hat{q}(D). \)
- Stay for the good signals where \( q(y) \geq q^D. \)
- Withdraw with probability \( r(y) \) for all intermediate signals.
This yields the following ex-ante participation constraint\(^\text{11}\)

\[
\int_0^{\hat{y}} pD f(y|g) dy + \int_{y_D}^{1} pD f(y|g) dy \\
+ \int_{\hat{y}}^{y_D} \frac{q(y)}{p f(y|g)} \left[ r^2 \lambda X I q(y) + r(1-r)I + (1-r)rq(y)D + (1-r)^2q(y)D \right] dy \\
\geq I
\]

which nicely shortens to \(pD \geq I\) due to our equilibrium refinement. Rewrite
the bank’s profit function

\[
\Pi^B = \int_0^{\hat{y}} 2p(XI - D) f(y|g) dy + \int_{y_D}^{1} 2p(XI - D) f(y|g) dy \\
+ \int_{\hat{y}}^{y_D} 2p f(y|g) \left[ r(1-r) \left( x \left( 2I - \frac{I}{\lambda X q(y)} \right) - D \right) + (1-r)^2(XI - D) \right] dy \\
= 2p(XI - D) - 2pD \left( \frac{1 - \lambda}{\lambda} \right) \int_{\hat{y}}^{y_D} r(y) f(y|g) dy
\]

Start with \(\lambda X p \geq 1\) such that both types of equilibria coexist. Therefore,
we have \(D \geq \lambda XI \geq \frac{I}{p}\). The first-order condition with respect to \(D\) is
(Remember, \(r(y^{D}) = 0\).)

\[
\frac{\partial \Pi^B}{\partial D} = -2p - 2p \left( \frac{1 - \lambda}{\lambda} \right) \int_{\hat{y}}^{y_D} \frac{I - 2Dq(y)}{I - \lambda X I q(y)} f(y|g) dy \\
+ 2pD \left( \frac{1 - \lambda}{\lambda} \right) \frac{I - \hat{q}D}{I - \lambda X I \hat{q}} \frac{\partial \hat{q}}{\partial D} f(\hat{y}|g) = 0.
\]

Due to the fact that the depositors withdrawal probabilities in the mixed
strategy zone depend on \(q(y)\), no simple expression of the optimal \(D^B\) can

\(^{11}\)Use \(\hat{y}\) to shorten \(q(\hat{y}) = \hat{q}(D)\) and \(r\) instead of \(r(y)\)
be offered unless we impose more precise assumptions on $f(y|·)$. Still, some results exist.

Suppose $D = \lambda XI$. Then, we have full withdrawal for all intermediate signals, and this interval of intermediate signals is largest. For all $D > \lambda XI$, the depositors’ mixed strategies lead to less frequent withdrawals and therefore some profits are left to a surviving bank. Moreover, the withdrawal interval of intermediate signals shrinks from both ends. All this comes at the cost of paying a higher $D$ in the good state. Thus, there is as tradeoff, strongly calibrated by $\lambda$. The banks profits increase if she sets $D > \lambda XI$ except for those cases where $\lambda$ is very high such that the effect via $2p(XI - D)$ dominates. Note that there is a non-linear dependency here.

Moreover, note that $D \geq \lambda XI$ reveals that $\Pi^B$ converges to zero if $\lambda \rightarrow 1$ since $D \rightarrow XI$. Thus, for large values of $\lambda$ we have $\Pi^A > \Pi^B$ independent of the distributions.

In those cases where $\lambda Xp < 1$, there is no equilibrium of type A but still there is an equilibrium of type B. Now we need to check for $D^B \geq \frac{I}{p} > \lambda XI$ or the zero-information participation constraint. For the very illiquid projects, $\lambda < \frac{1}{2pX-1}$, we now get $D^B = \frac{I}{p}$ and no withdrawals at all since $q^D = p < \hat{q}(D)$. For more liquid projects, we have $D^B > \frac{I}{p}$ by the same arguments of shrinking the withdrawal interval and mixed-strategy withdrawals.

**Proof to Proposition 2.** Comparing the first and the second constraint of the above maximization problem, we get that the ex ante participation constraint is binding if

$$D^{part} \geq \frac{I}{q_2}.$$
This holds if and only if

\[ \omega \geq \frac{p}{1 - p} (\lambda X - 1) (1 - \gamma) - 1. \]  

(8)

Finally observe that \( D^{part} < \lambda X I \) is equivalent to \( \lambda p X > 1 \).

QED

8 Add ons

8.1 Right to withdraw boundary \( y < Y \)

Suppose the bank and the depositors contractually agree that depositors are free to withdraw for sufficiently bad new, \( y < Y \), only. To stay in line with the model assumption about bargaining power so far, we assume that the bank is able to make a take it or leave it offer about \( Y \) as well as \( D \).

Consider the effects on equilibrium A first where depositors withdraw for all \( y \leq y^D \) and stay otherwise. If the agreement is such that \( Y \geq y^D \), there is no effect. If \( Y < y^D \) instead, there is an area \( [Y; y^D] \) where depositors would like to withdraw, but are not allowed to do so. This effect immediately enters the participation constraint of the depositors. For not being able to do what they prefer, they need to be compensated with a higher \( D^{Part} \). The optimal value of \( D \) is either determined by the first-order condition, \( D = D^* \) with \( \frac{\partial \Pi^A}{\partial D} = 0 \), or by the participation constraint, which yields \( \frac{\partial \Pi^A}{\partial D} < 0 \) joint with
\( \frac{\partial D^{\text{part}}}{\partial Y} > 0 \). Looking for the profit maximizing \( Y \) now yields

\[
\frac{\partial \Pi^A}{\partial Y} = \frac{\partial \Pi^A}{\partial D} \frac{\partial D^A}{\partial Y} - 2pf(Y|g)(XI - D) + 2I \left( \frac{1}{\lambda q(Y)} \right) Xpf(Y|g)
\]

\[
= \frac{\partial \Pi^A}{\partial D} \frac{\partial D^A}{\partial Y} < 0 \quad \text{since} \quad q(Y) < q^D
\]

\[
< 0.
\]

and the bank sets \( Y \) as low as possible.

For \( Y < y^* \) the problem changes to

\[
\max_{D,Y} 2p(1 - F(Y|g))(XI - D),
\]

with \( \frac{\partial \Pi^A}{\partial D} = -2p(1 - F(Y|g)) < 0 \) such that the participation constraint will be binding.

\[
\frac{\partial \Pi^A}{\partial Y} = \frac{\partial \Pi^A}{\partial D} \frac{\partial D^{\text{part}}}{\partial Y} < 0
\]

yields \( Y = 0 \) and \( D^A = \frac{I}{p} \). Withdrawals are prohibited for all signal realizations.

Consider equilibrium B now. Note first that setting \( Y > y^D \) again has no effect. Similarly, offering \( Y < \hat{y} \) doesn’t have an effect as depositors do not want to withdraw anyway. Depositors behavior is affected if \( Y \in (\hat{y}; y^D) \).

Optimizing over \( Y \) now yields

\[
\frac{\partial \Pi^B}{\partial Y} = \frac{\partial \Pi^B}{\partial D} \frac{\partial D^B}{\partial Y} < 0
\]

\[
-2pD\frac{1 - \lambda}{\lambda} r(Y)f(Y|g) < 0.
\]
As under equilibrium A, we have $\partial \Pi_B \partial D = 0$ if $D^B = D^\ast$. If $D^B = D^{Part}$ instead, we get $\frac{\partial \Pi_B}{\partial D} < 0$ and $\frac{\partial D^{Part}}{\partial Y} > 0$. Thus, the first-order derivative is strictly negative. The bank chooses $Y$ as low as possible. The bank offers either $Y = \hat{y}$ or $Y = 0$ as there is no difference.

Taking these results together, the bank will set $Y$ such that withdrawals will never occur.

Intuition: As we noted at the outset, the bank would strictly prefer not to provide any information to depositors in our setting. Nevertheless, we assume that this is not allowed exogenously. Setting a limit $Y$ such that depositors are allowed to withdraw for signal realizations worse than the limit, $y < Y$, is essentially an instrument to make communication worthless. If the bank has to communicate the signal, but is able to prohibit reactions, this is the same as not offering any information. Thus, the bank sets $Y$ such that she prevents depositors from reacting. Through the back door, we reestablished the no-(valuable)-information setting. Thus, by the same reason that we call information provision indispensable, we need to protect depositors from the adhesion contracts the bank would like to implement.
References


