Private Benefits of Control, Mandatory Disclosure, and the Choice Between Public and Private Debt *

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Abstract

This study analyzes the effects of private benefits of control on the optimal quality of mandatory disclosure in a debt financing setting. We provide a model in which a manager with private benefits chooses between a private and a public debt market, with private lenders being superior in their monitoring capabilities. Thus, firms' debt market choices are not only affected by markets' differing informational needs but also managers' incentives to avoid losing control. Mandatory disclosure prevents lenders from funding unfavorable projects and efficiently allocates firms into private and public debt markets. However, we show that mandatory disclosure can also work as a catalyst that allows managers to access public bond markets and secure their private benefits, although the private debt market would be socially desirable. The positive effects of higher disclosure quality outweigh this cost when private benefits are weak or strong but enable managers to act on their tendency to secure private benefits when being medium.

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1 Introduction

This study analyzes the impact of private benefits of control on firms' debt market choices and the implications for mandatory disclosure. Our research is motivated by the well-established view that private lenders are, compared to public investors, superior monitors that can overcome coordination problems that impede monitoring and efficient liquidation (e.g., Diamond, 1984; Rajan, 1992). Under private benefits, managers' debt market choices are not only affected by the markets' differing informational needs but also by the desire to stay in control. We show that mandatory disclosure can work as a catalyst allowing managers to access public bond markets while at the same time securing their private benefits. We determine the socially optimal level of mandatory disclosure and explain why firms may not make socially optimal financing choices when considering private benefits, how private benefits affect the optimal mandatory disclosure level, and why high-quality mandatory disclosure may not be socially optimal.

In our model, firms are capital constraint and need to raise debt capital to fund an investment project. Firms are managed by their owners (Diamond, 1984; Holmstrom & Tirole, 1997; Rajan, 1992) that enjoy non-monetary benefits from staying in control (Aghion & Bolton, 1992).[1] Managers can access either the private or the public debt market. Private lenders monitor each loan contract and can efficiently liquidate a firm. Investors in the public bond market abstain from monitoring due to coordination problems. Firms are further subject to mandatory disclosure and provide a noisy signal about their project's success probability.

We show that increasing the quality of disclosures can have positive or negative effects on social welfare in such a setting. The positive effects arise from lenders' ability to price debt more efficiently and constrain the funding of value-destroying projects. The negative effect is an unattended consequence of the more efficient pricing. Because public lenders cannot overcome information asymmetries after initiation, they react stronger to reliable more good news than their private lender counterparts. This allows managers to enter the public market and protect their private benefits of control even if utilizing private

[1] Another interpretation is that managers' private benefits are proportional to the funds under management (e.g., Holmstrom & Tirole, 1997).

lenders' monitoring abilities is socially desirable.

We find that the positive effects outweigh the unintended consequences when private benefits are weak or strong, but not for medium levels. If private benefits are weak, managers focus on the firm value and are unlikely to choose inefficient debt financing to stay in control. If private benefits are strong, managers always prefer public debt, making a higher disclosure quality desirable to discipline managers' financing choices for bad news. However, for medium levels of private benefits, higher quality disclosures can allow managers to act on their proclivity to avoid private lenders' monitoring. The public debt market reacts stronger to good news, which reduces the manager's cost of protecting private benefits. In sum, our paper highlights that firms' financing choices and more importantly, the optimal disclosure quality depends on managers' private benefits. Requiring an ever-increasing disclosure quality may not be socially desirable in such a setting.

This paper contributes to the literature's understanding of private benefits' effects on firms' debt market choices. Based on the assumption that private lenders are superior monitors compared to public investors (e.g., Diamond, 1984; Rajan, 1992), explicitly considers the connection between firms' debt financing choices and managers' expected private benefits based on the differences in debt markets' monitoring capabilities. Similar to Almazan and Suarez (2003), our model does not rely on private lenders' advantage in evaluating firms' profitability, but on that they perform monitoring that managers dislike. However, we further consider mandatory disclosure and its interaction with managers' private benefits. By doing so, our model combines prior monitoring models (e.g., Holmstrom & Tirole, 1997; Rajan, 1992) with models considering private benefits (e.g., Aghion & Bolton, 1992).

This study further contributes to the literature examining the relation between disclosure and firms' debt. Private lenders, who are mostly financial intermediaries, have an advantage in monitoring borrowers compared to public debt markets. This implies that firms in a poor information environment will more likely raise funds in private debt markets because of lower adverse selection costs (e.g., Bharath, Sunder, & Sunder, 2008; Diamond, 1984). A strong information environment mitigates adverse selection in the public bond market, making mandatory disclosure a commonly advocated solution. Our model confirms this notion but also highlighting a potential adverse effect. We show that it may not be socially optimal to mandate higher levels of disclosure under private benefits. Mandatory disclosure can incline managers to enter the public bond market, although the private debt market with its superior monitoring may offer cheaper debt capital. Private benefits result in debt market choices with higher than socially desirable financing costs and less efficient monitoring.

This paper proceeds as follows. In Section 2, we introduce the model and in section Section 3 we provide the analysis of the model. Section 4 concludes.

2 Model

The model economy consists of three risk-neutral players: a manager (M), a private lender (L), and a public lender (P). The manager has no wealth and controls a firm that requires capital of $K > 0$ to undertake a risky project.

The model has four dates. At date 0, the firm is mandated to release a public signal that is informative about the project's profitability. At date 1, the manager offers a debt contract to lenders in the private and public debt market. The contract includes a covenant that may allow lenders to assume control during the lending term. If one of the lenders provides financing, the manager pursues the project. Otherwise the game ends. At date 2, the private lender may acquire an additional signal by monitoring the project, whereas the public market faces prohibitively high coordination and monitoring costs (Diamond, 1984; Roberts & Sufi, 2009). If an unsuccessful project is uncovered, the lender assumes control via the covenant and liquidates the project. Finally, at date 3, cash flows of continued projects realize.

2.1 Project

The project succeeds (S) with probability $\tilde{p}$ and fails (F) with probability $1 - \tilde{p}$. It is common knowledge that $\tilde{p}$ is uniformly distributed between zero and one, $\tilde{p} \sim U[0, 1]$, with mean $E[\tilde{p}] = \bar{p} = 1/2$. The realization of p is ex-ante unknown to all players. We refer to the realization p as project type. At date 2, a successful project yields a higher cash flow than an unsuccessful project, $X_S > X_F \geq 0$. Without loss of generality, we normalize the cash flow of a failed project to zero, $X_F = 0$, and assume that the expected ex-ante net present value (NPV) of the project is zero, i.e., $\bar{p}X_S = X_S/2 = K$.[2]

2.2 Mandatory disclosure

Firms are subject to mandatory disclosure, resulting in a noisy signal $s \in [0, 1]$ about the project's type. Disclosure occurs at date 0 before the manager attempts to raise debt capital. The signal matches the project type, $s = p$, with probability $q \in [0, 1]$, and is uninformative with probability $1 - q$. In the latter case, the signal is randomly drawn from the distribution of $\tilde{p}$. We refer to q as signal quality or precision. To focus on the informational role of the signal, we assume that the signal is costless and that disclosure is truthful. We discuss the effect of disclosure costs in Section 3.4. Upon disclosure of s, all players form Bayesian beliefs about the project type, which is a weighted average of the prior and the signal:

$$\pi(q) \equiv E[p \mid s, q] = qs + (1 - q)\bar{p} = \frac{1}{2} + q\left(s - \frac{1}{2}\right). \tag{1}$$

When signal quality increases, players assign increasing weight to the signal and decreasing weight on the prior.

2.3 Debt financing

At date 1, the manager offers a 'take it or leave it' debt contract to potential lenders. The manager has all bargaining power, which is equivalent to assuming that both the public debt market and a private debt market are prefectly competitive. The debt contract is the same for both markets, and includes direct lending costs and a covenant.[3] Direct lending costs are represented by a zero-coupon debt instrument with face value D_i, where

[2]The results do not qualitatively change when assuming a positive or negative NPV.

[3]Since the manager has all bargaining power and can perfectly anticipate lenders' acceptance decisions, limiting the analysis to a single contract is without loss of generality.

$i \in \{L, P\}$. Lenders have a claim to the project's cash flow up to its face value.[4] We restrict attention to $X_S \geq D_i \geq K$. For $D_i > X_S$, lenders know that the manager would never be able to fully meet such contractual obligations.[5] For $D_i < K$, lenders do not provide capital because they are never fully repaid. The manager always includes a covenant in the contract offer. He ex-ante prefers committing to assign decision rights to the lender if the lender's monitoring produces a signal indicating that the project will fail.[6]

Public market

The public market consists of an infinite number of widely dispersed arm's-length investors with identical preferences. We collectively refer to these investors as the public lender (P). The public lender neither monitors nor interferes with the project prior to completion, reflecting the free-riding problems arising from the large number of investors (Diamond, 1984; Roberts & Sufi, 2009). Providing financing yields an expected payoff of

$$EU_P = E[p|s, q]D_P - K. \tag{2}$$

Private market

The private market consists of financial intermediaries, such as banks or loan syndicates, which we collectively refer to as the private lender (L). Relative to the public market, there is a cost and a benefit to private lending. On the one hand, the private lender's capital supply is limited, resulting in capital rationing costs of gK, with $g > 0$, and the overall lending costs being $(1 + g)K$. On the other hand, the private lender is a superior monitor who can overcome the public market's coordination problems (Diamond, 1984; Roberts & Sufi, 2009). As such, the private lender may engage in costly monitoring after debt initiation to learn the project's future realization. Specifically, the lender sets the probability of perfectly learning the project's future cash flow realization, $m \in [0, 1)$, at a

[4]A higher difference between D_i and K reflects a higher implicit interest rate.

[5]All contracts with $D_i > X_S$ are equivalent to $D_i = X_S$.

[6]We verify this assumption in the proof of Lemma 1.

cost of $C(m) = 0.5c_m m^2$ that is convex in m, to maximize the expected payoff

$$EU_L = E[p \mid s, q]D_L - (1+g)K + m\left(1 - E[p \mid s, q]\right)\alpha K - \frac{c_m}{2}m^2. \tag{3}$$

If the private lender learns that the project will be unsuccessful, she assumes control via the covenant and liquidates the project. We assume that a liquidation value of αK can be recovered, where $\alpha \in (0, 1)$ reflects the costs of the early intervention.[7] To ensure an interior solution, we assume that the marginal monitoring costs are sufficiently high relative to the liquidation value, $\alpha K < c_m$. Furthermore, we assume that capital rationing costs are not so high that public lending strictly dominates private lending, i.e., $0 < g < \frac{\alpha^2 K}{8c_m}$.

Manager The manager is the residual claimant and receives all cash flows exceeding the face value D_i at date 3. Additionally, he obtains non-monetary private benefits of $B \geq 0$ for staying in control of the project until completion, where B is independent of the project's success.[8] Private benefits do not reduce the project's value and cannot be transferred to third parties because they non-monetary and not verifiable (Aghion & Bolton, 1992). An alternative interpretation is that the manager enjoys a continuing stream of private benefits proportional to the funds under management.

The manager sets D_i and makes his financing choice to maximize his expected payoff. The manager's action set is $A = \{\emptyset, P, L\}$, where $\emptyset$ denotes not obtaining financing, P entering the public market, and L entering the private market.

The payoffs for each financing choice are

$$EU_M(\emptyset | s, B) = 0, \tag{4}$$

$$EU_M(P | s, B) = E[\tilde{p} \mid s](X_S - D_P) + B, \tag{5}$$

$$EU_M(L | s, \hat{m}, B) = E[\tilde{p} \mid s](X_S - D_P) + (1 - \hat{m}(1 - E[\tilde{p} \mid s]))B, \tag{6}$$

[7]Without a covenant in place, public lending strictly dominates private lending. The private lender has no monitoring incentives such that private lending has the same features as public lending except for higher lending costs due to capital rationing.

[8]As in Caskey and Laux (2017), we obtain qualitatively similar results assuming $B_S > B_F > 0$.

where $\hat{m}$ is the manager's conjecture about the private lender's monitoring effort.

3 Analysis

3.1 Equilibrium

We begin by characterizing the general equilibrium of the model using backward induction. Lemma 1 summarizes the equilibrium strategies of the lender in both debt markets after observing the signal s about the project type. The following Lemma summarizes the strategies.

Lemma 1. *After observing signal s with signal quality q,*

1. *the public lender accepts a contract if*

$$D_P\left(q,s\right) \geq \underbrace{\frac{1}{E[p \mid q,s]}}_{\textit{risk premium}} K = \underline{D}_P \tag{7}$$

and rejects the contract otherwise, and

2. *the public lender accepts a contract if*

$$D_L\left(s,q\right) \geq \frac{1}{E[p \mid q,s]}\left(K + \underbrace{gK}_{\substack{\textit{capital}\\ \textit{rationing costs}}} - \underbrace{\frac{(1-E[p \mid q,s])^2\alpha^2K^2}{2c_m}}_{\textit{monitoring benefit}}\right) = \underline{D}_L \tag{8}$$

and rejects the contract otherwise.

Proof: See appendix.

First, consider the public lender. Since the public market is perfectly competitive, the public lender accepts all debt contracts such that he at least breaks even, i.e. satisfying (2) ≥ 0. Rearranging the break even condition yields the face value D_P in Lemma 1. To ensure break even, the manager has to offer a debt contract that price protects the lender against a default. Therefore, the face value contains a risk premium (RP) on the capital invested based on the information about the project type. For instance, a bad signal, $s < 1/2$, indicates that the project is a bad type rather than a good type project which means that after observing a bad signal, lenders face a higher risk of default compared to a good signal. To ensure that the lender breaks even in expectation, the manager therefore offers a higher risk premium for bad signals than for good signals.

Next, consider the private market. Parallel to the public market the lender accepts all debt contracts satisfying (3) ≥ 0 due to the competitive market assumption. Rearranging the break even condition yields the face value D_L in Lemma 1. Again, the manager offers a risk premium to price protect the lender against a default, yet, the premium is calculated on a different basis. In addition to the capital invested, there are two further components: capital rationing costs (CRC) and benefits from monitoring (MB). Capital rationing costs linearly increase the capital invested, and thus the face value, but are independent of the project type. With regards to monitoring, private lenders can benefit from monitoring because they can liquidate an unsuccessful project early. After entering the debt contract, the private lender's expected payoff is given by (3). At this stage, the lender maximizes the expected payoff by choosing a monitoring effort

$$m^* = \frac{(1 - E[p \mid q, s])\alpha K}{c_m}. \tag{9}$$

The optimal monitoring effort trades off the expected benefits from early liquidation given the information available with effort costs of monitoring. Since bad signals indicate a higher probability of default, lenders choose a higher monitoring effort after observing a bad signal compared to a good signal. In our model, monitoring always adds value and therefore reduces the face value.

In equilibrium, the manager anticipates the lenders' strategies given the signal including the private lender's monitoring choice and enters the debt market that maximizes his expected payoff. For this purpose, he offers a face value so that the respective lender's break even condition becomes binding. Before we continue to discuss the manager's financing choice in equilibrium, we elaborate on the impact of signal quality on the minimum face values for which lenders accept financing.

Corollary 1. *For good signals, $1/2 < s \leq 1$, the minimum face values in the public market, $\underline{D}_P$, and the private market, $\underline{D}_L$, are decreasing in signal quality. For bad signals, $0 \leq s < 1/2$, the minimum face value in the private market, $\underline{D}_L$, is increasing in signal* *quality.*

Proof: See appendix.

Increasing signal quality has a differential effect on the minimum face value for good versus bad signals. Recall that with increasing signal quality, lenders place less weight on the prior and more weight on the signal. That means, a higher signal quality corresponds to a higher correlation between the signal and the project type. For a given good signal, the lender becomes more confident that the project is of a good type as the signal quality increases, and thus, the lender expects that a default is less likely. For a bad signal, the reverse holds, so that the lender expects a higher chance of default.

Consider first the impact of signal quality on the minimum face value in the public market:

$$\frac{\partial \underline{\mathrm{D}}_P(q,s)}{\partial\, q} = \frac{\partial RP(q,s)}{\partial q} K \tag{10}$$

Here, only the risk premium depends on the signal and therefore also on the signal quality whereas the capital invested is independent of the signal. Any changes in risk are reflected in the risk premium, and consequently, the face value increases (decreases) in signal quality for bad (good) signals.

The minimum face value in the private market additionally consists of the monitoring benefit and capital rationing costs:

$$\frac{\partial \underline{\mathrm{D}}_L(q,s)}{\partial\, q} = \frac{\partial RP(q,s)}{\partial q} \big(K + gK + MB(q,s)\big) + RP(q,s) \frac{\partial MB(q,s)}{\partial q} \tag{11}$$

The signal quality has a direct impact on the monitoring benefit because it alters the lender's monitoring choice. Capital rationing costs, in contrast, are independent of the signal and signal quality. Lastly, the indirect effect via the risk premium affects each of the face value components.

To illustrate the direct effect of signal quality on the monitoring benefit, consider the case where there is a perfectly uninformative signal, i.e. $q = 0$. In this case, the private lender chooses a constant monitoring effort of

$$m^*(q=0) = \frac{\alpha K}{2c_m}$$

for all signal realizations. However, when the signal is informative, a good signal indicates a lower chance of a default which makes monitoring less valuable. Consequently, the lender has less incentives to monitor the project. The reverse holds for a bad signal, where a higher risk of default makes monitoring more valuable resulting in a higher monitoring effort. As the signal becomes more precise, the lender becomes more confident that the signal represents the true project type which decreases (increases) the value of monitoring for good (bad) signals, and in response, decreases (increases) the incentives to monitor for the lender. Since monitoring always adds value for the lender, a decrease (increase) in monitoring benefit corresponds to an increase (decrease) in face value.

In sum, we have two effects of signal quality which run in opposite directions: The direct effect on the monitoring benefit and the indirect effect on the risk premium. The risk premium effect is clearly dominating because it has a bigger leverage, the capital invested K, compared to the leverage of the monitoring effect, liquidation value $\alpha K < K$. However, as we will discuss later, the monitoring benefit effect plays an important role in determining the manager's optimal financing choice.

In the next step, we consider the financing options of the manager, which we summarize in Lemma 2.

Lemma 2. *The manager's set of financing options for a given signal s are*

$$\begin{array}{ll} \{\emptyset\} & \text{if } s < s_L(q) \\ \{\emptyset,\, L\} & \text{if } s_L(q) \leq s < s_P \\ \{\emptyset,\, L,\, P\} & \text{if } s_P \leq s \end{array}$$

where $s_L(q) = \frac{1+q}{2q} - \frac{4c_m - \Omega}{2q\alpha^2 K}$, with $\Omega = \sqrt{8c_m\left(2c_m - (1-g)\alpha^2 K\right)}$ and $s_P = 1/2$ derive from the lenders' financing conditions, and $s_L \geq 0$ for $q \geq q_L = (4cm - \Omega)/(\alpha^2 K) - 1$.

Proof: See appendix.

The lenders' financing constraints determine the manager's financing options. From Corollary 1 it follows that face values decrease in signal quality for good signals but increase for bad signals. Eventually, the face value becomes larger than the cash flow of a successful project for some very bad signals. In this case, the lender declines the financing because she makes a loss in expectation. Considering the private market, this happens when the signal becomes sufficiently precise, i.e., $q > q_L$, such that the private lender only accepts the debt contract if the signal is sufficiently good, $s_L(q) \leq s < 1/2$, and declines the debt contract otherwise. In the public market, the constraint is independent of the signal quality. In particular, the public lender declines a debt contract for all bad signals, i.e. $s < 1/2$. In consequence, the manager can only choose between the public and the private market after observing a good signal, $s \geq 1/2$. The subsequent Corollary summarizes the key insight from Lemma 2.

Corollary 2. *If the public market is available, the private market is also available but not vice versa.*

Proof: Follows from Lemma 2.

The Corollary underlines that obtaining financing for the project is more difficult in the public market rather than in the private market because the public market lacks the possibility of early intervention. However, Lemma 2 only provides the manager's financing options but not the actual financing choices. In equilibrium, the manager chooses the debt market that is more attractive in terms of financing conditions, resulting in the following characterization of the manager's equilibrium strategy.

Proposition 1. *In equilibrium, the manager conjectures $\hat{m} = m^*$, and depending on signal quality q and private benefits B obtains,*

(a) *no financing if* $0 \leq s < s_L(q)$

(b) *financing in the private market with* $D_L = \underline{D}_L$ *if* $s_L(q) \leq s < \max(s_P, s_I(q, B))$*, and*

(c) *financing in the public market with* $D_P = \underline{D}_P$ *if* $\max(s_P, s_I(q, B)) \leq s \leq 1$.

where $s_I(q, B) = 1/2 + 1/(2q)\left(1 - \sqrt{\Phi(B)}\right)$ *with* $\Phi(B) = (8gc_m)/(\alpha(\alpha K - 2B))$ *derives from the manager's indifference condition and* $s_I(q, B) \leq 1$ *for* $q \geq 1 - \sqrt{\Phi(B)} = q_I$.

Proof: See appendix.

The proposition follows from the manager's profit maximization strategy in combination with Lemma 1 and 2. The manager always prefers to conduct the project because there is a strictly positive chance of a positive payoff if the project succeeds and no downside if the project fails due to limited liability. Thus, the manager will always obtain financing for the project given that lenders accept the debt contract. For bad signals the only available financing source is the private market. For good signals, in contrast, the manager chooses the financing source which maximizes his expected payoff. The following condition formalizes the manager's choice

$$E[p \mid q,s](X_S - D_L) + (1 - m(1 - E[p \mid q,s]))B \geq E[p \mid q,s](X_S - D_P) + B \tag{12}$$

At $s_I(q,B)$ the indifference condition becomes binding yielding the last part of the Proposition. Unlike the lenders' strategies, the manager's equilibrium strategy depends on his private benefits. We will discuss the impact of private benefits on the equilibrium behaviour of the manager in the subsequent sections. Corollary 3 describes how the thresholds $s_L(q)$ and $s_I(q,B)$ from Proposition (1) change in the signal quality.

Corollary 3.

For all q

$$\frac{\partial\, s_L(q)}{\partial\, q} > 0 \quad \text{and} \quad \frac{\partial\, s_I(q,B)}{\partial\, q} < 0 \tag{13}$$

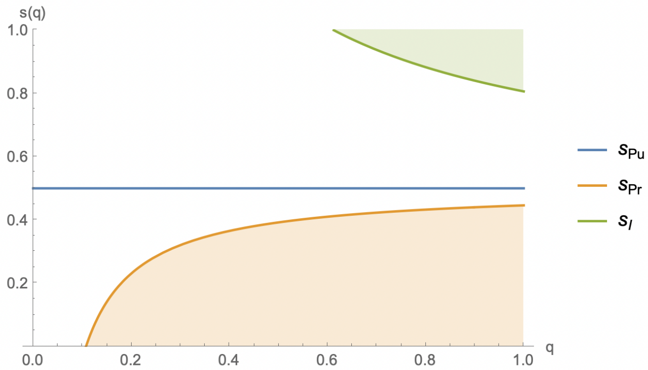
Proof: See appendix.

Following Corollary 1, the face value in the private market is strictly increasing in signal quality for bad signals. Consequently, the threshold for signals for which the manager will not obtain financing $s_L(q)$ is increasing in signal quality as well. That is, the interval (a) in Proposition 1 becomes larger. Moreover, the face value is decreasing in signal quality for high signals in both markets, yet, at a different rate. Compared to the public market, face value in the private market is less sensitive to a change in signal quality because monitoring has a moderating effect. Thus, the face value in the public market is decreasing at a higher rate for $s < s_I(q,B)$, implying that the public market becomes more attractive with increasing signal quality. This, in turn, means that $s_I(q,B)$ is decreasing in signal quality and that the interval (c) becomes larger as well. Lastly, as intervals

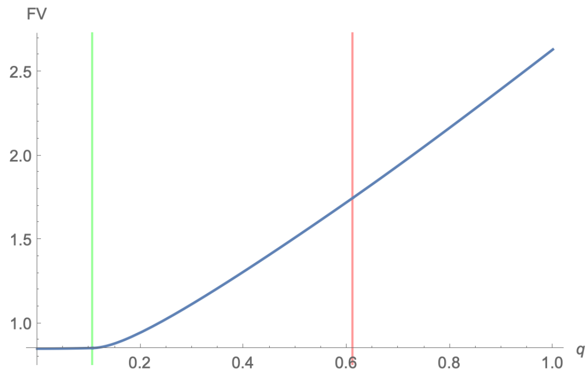
(a) and (c) are increasing, it follows that interval (b) must be decreasing such that the number of signals for which the manager obtains financing in the private market decreases in signal quality.

3.2 Benchmark – Absence of private benefits

We next characterize the impact of signal quality on firm value (V). We begin our analysis with a benchmark setting in which we mute the manager's private benefits. There is no conflict of interest because the manager only cares about firm value.



(a) Debt market choice as a function of signal realization and signal quality.



(b) Expected firm value as a function of signal quality. The green line indicates q_L and the red line q_I.

Figure 1: Debt market choice and expected firm value absent private benefits. Parameter values: $K = 10$, $\alpha = 0.8$, $g = 0.01$, and $c_m = 8$.

Regardless of the private benefits, the lenders' strategies are given by Lemma 1 and the manager's strategy derives from Lemma 2, with the manager maximizing firm value by choosing the debt market with the lowest face value. Figure 1a (a) illustrates the debt market choices of the manager without private benefits for a given signal realization s and signal quality q. The orange area indicates signal realizations for which the manager does not obtain financing. The white captures parameter constellations for which the manager prefers the private over the public debt market. The green area indicates signal realizations for which the manager prefers the public market over the private market. Given the manager's equilibrium strategy we can derive the firm value as well as the impact of signal quality on firm value in the benchmark case.

Proposition 2. *Absent private benefits, $B = 0$, the expected firm value is strictly increasing in q.*

Proof: See appendix.

To separate the economic forces, consider a case in which the signal does not convey any information, i.e., $q = 0$. In this case, all players stick with their prior expectations about the project type for all signals. Hence, the risk premium is constant for all signal realizations. However, unlike the public lender, the lender in the private market receives monitoring benefits in addition to the capital invested. Absent further information, these monitoring benefits outweigh the higher costs from capital rationing so that the manager obtains a lower face value in the private market compared to the public market. As such, the manager obtains financing in the private market for all signals and the firm value amounts to

$$\begin{aligned}&\int_0^1 V(L \mid q, s, m^*, B = 0)\, ds = \\ &\qquad \int_0^1 \left(E[p \mid s]2K - (1+g)K + \frac{(1 - E[p \mid s])^2 \alpha^2 K^2}{2c_m} \right) ds\end{aligned} \tag{14}$$

An increase in signal quality allows the manager to offer face values based on the additional information contained in the signal. From Corollary 1 it follows that an increase in signal quality causes an increase of the face value for bad signals. For very low levels of signal quality, i.e., $q \in (0, q_L)$, however, the manager still obtains financing for all signals in the private market because the face value remains sufficiently low. For good signals the face value decreases in signal quality whereby the rate of change differs for the private and public market. Yet, for low levels of signal quality, the change in the face values is insufficient to lure the manager into the public market even for the best signal. Consequently, the private market remains the only source of financing for the manager.

Consequently, the only effect on firm value results from monitoring which becomes more efficient as signal quality increases. In particular, more efficient monitoring increases the monitoring benefit for bad signals but decreases the benefit for good signals, whereby the former effect dominates the latter. Now, since the private debt market is competitive the private lender demands a lower interest rate as the monitoring benefit increases. That means, any efficiency gain from monitoring accumulates to the manager resulting in an increase in firm value.

When signal quality reaches q_L, the face value for the worst signal, $s < s_L(q)$, exceeds the cash flows from the project so that lenders deny financing. The calculation of the firm value changes as follows:

$$\int_{s_L(q)}^{1} V(L \mid q, s, m^*, B = 0)\, ds =$$

$$(14) - \underbrace{\int_0^{s_L(q)} \left[\underbrace{\left(E[p \mid s]2K - K \right)}_{PR} + \underbrace{\left(\frac{(1 - E[p \mid s])^2 \alpha^2 K^2}{2c_m} \right)}_{MB} - \underbrace{\left(gK \right)}_{CRC} \right] ds}_{investment\ efficiency} \quad (15)$$

We refer to this effect as "investment efficiency". An increase in signal quality affects investment efficiency, and thus, face value in three ways. First, consider the return of the project (PR) regardless of financing effects. Recall, that lenders decline financing only for those signals that very likely come from negative NPV projects. Since negative NPV projects have an adverse effect on firm value, sorting them out increases firm value in expectation. With better information, lenders can identify negative NPV projects more precisely so that this effect strictly increases firm value. Second, declining financing for *any* signals results in a loss of monitoring benefits. For bad signals, this loss is larger than for good signals because monitoring is more valuable for the former. Moreover, the effect becomes even stronger as signal quality increases. Hence, the monitoring benefits component (MB) is always negative. Third, the manager saves capital rationing costs for those signals for which he does not obtain financing. Consequently, this capital rationing cost effect (CRC) also has a positive effect on firm value. As the project return effect dominates the monitoring effect, the total effect of investment efficiency on firm value must be positive for all q.

Lastly, as signal quality further increases, the public market becomes more attractive so that, for $q_L < q_I < q < 1$, the manager prefers the public market for the best signals $s > s_I(q)$. In this case, the firm value is determine as follows.

$$\int_{s_L(q)}^{s_I(q)} V(L \mid q, s, m^*, B=0)\, ds + \int_{s_I(q)}^{1} V(P \mid q, s, B=0)\, ds =$$

$$(15) - \underbrace{\int_{s_I(q)}^{1} \left[\underbrace{\left(\frac{(1 - E[p \mid s])^2 \alpha^2 K^2}{2c_m} \right)}_{MB} - \underbrace{\left(gK \right)}_{CRC} \right] ds}_{\textit{financing efficiency}} \qquad (16)$$

While investment efficiency is about *which* projects should obtain financing, financing efficiency is about *in which market* projects should obtain financing. Again, we can decompose the impact of increasing signal quality on financing efficiency and firm value into two effects. First, switching from the private to the public market implies a loss of monitoring benefits which decreases firm value. From Corollary 3, it follows that with increasing signal quality the number of signals for which the manager obtains financing in the public market increases. Consequently, monitoring benefits are forgone more often but at the same time the loss of monitoring benefits becomes less severe because monitoring benefits are decreasing in signal quality for good signals. Since the former indirect effect dominates the latter direct effect, the net effect of monitoring benefits on firm value is negative. Second, in the public market, the manager faces no capital rationing costs. With increasing signal quality the manager has to pay these costs less often rendering the impact on firm value positive.

Absent private benefits, the net effect of financing efficiency is positive because the capital rationing costs effect dominates the monitoring benefits effect for all levels of signal quality. These findings together with the findings on investment efficiency yields Proposition 2. Figure 3 provides an illustration of Proposition 2.

In the benchmark setting, more information in the sense of increasing signal quality is valuable because it facilitates sorting out negative NPV projects (investment efficiency) and trading-off monitoring benefits versus capital rationing costs (financing efficiency).

3.3 Presence of private benefits

We next unmute the managers' private benefits and assume $B > 0$. As a consequence, the manager may prefer to enter the public bond market to secure private benefits in equilibrium, which does not maximize firm value as private benefits are non-monetary.

For bad signals, the strategies of both the lenders and the manager remain unaffected by the manager's private benefits. For good signals, however, private benefits play an important role because they alter the manager's debt market preferences which, in turn, affects firm value. The following Lemma summarizes the impact of private benefits on the manager's equilibrium debt market decision.

Lemma 3. *For* $\overline{B} \equiv \dfrac{\alpha K}{2} - \dfrac{4gc_m}{\alpha} > B > 0$, $\dfrac{\partial s_I(q,B)}{\partial B} < 0$ *and* $s_I < 1/2$. *For* $B > \overline{B}$, $s_I = 1/2$.

Proof: See appendix. Private benefits provide additional incentives for the manager to

enter the public debt market. In particular, the manager accepts higher face values in the public market compared to the benchmark case, resulting in a lower $s_I(q, B)$. The red area in figure 2 indicates the additional signals for which the manager enters the public market for different levels of private benefits. An increase in private benefits relates to a shift of the threshold to the left highlighting that the manager enters the public more often compared to the benchmark case. As private benefits approach $\overline{B}$, the manager chooses the public market whenever the public market is available, i.e., for all $s \geq s_P$.[9]

Given the equilibrium strategies, we can characterize the impact of signal quality on firm value for different levels of private benefits.

Proposition 3. *Depending on the size of private benefits, firm value ambiguously relates*

[9]Note, that this threshold stems from the lenders' financing constraint but not from the manager's indifference condition. For very high private benefits, $B > \overline{B}$, the manager would prefer entering the public market also for signals $s < s_P$.

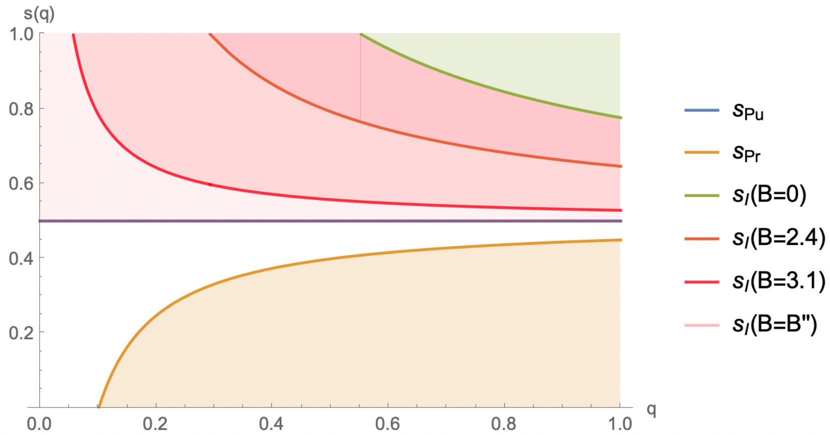


Figure 2: Debt market choice as a function of signal realization and signal quality with $B = 0$, $B = 2.4$, $B = 3.1$ and $B = \overline{B}$. Parameter values: $K = 10$, $\alpha = 0.8$, $g = 0.01$, and $c_m = 8$

to a change in signal quality, q. There exist two unique thresholds, $0 < \underline{B} < \overline{B}$, such that for

i. low private benefits, $0 < B < \underline{B}$, firm value is strictly increasing in q,

ii. intermediate private benefits, $\underline{B} < B < \overline{B}$, firm value is increasing in q for $0 < q < q_I(B)$ and U-shaped in q for $q_I(B) < q < 1$,

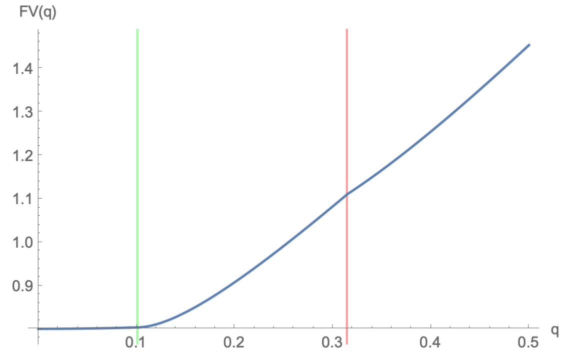
iii. for high private benefits, $\overline{B} < B$, firm value is strictly increasing in q.

Proof: See appendix.

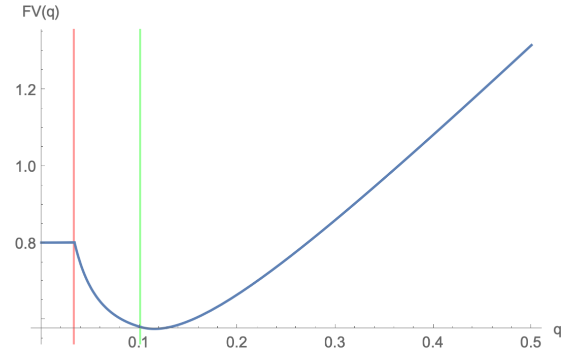
Private benefits do not alter the equilibrium strategies for bad signals, because the manager does not have a viable choice between public and private debt. The lenders effectively determine whether or not the project is funded, which means that investment efficiency must have a positive effect on firm value in this setting as well. In contrast, the effect on financing efficiency is a priori unclear because private benefits distort the manager's financing decision when he does have a choice between public and private debt.

To gain a better understanding of the impact of private benefits on firm value, let us assume that the signal quality is fixed and recall that financing efficiency consists of two components: capital rationing costs and monitoring. From Lemma 3 it follows that, all else equal, the manager prefers the public market more often when private benefits are higher. As a result, the manager pays capital rationing costs less often compared to the benchmark case, which always increases firm value. Hence, private benefits do not affect the direction of the capital rationing effect but the magnitude. Concerning the monitoring effect, entering the public market more often is associated with a loss of monitoring benefits, and thus, a decrease in firm value. Compared to the benchmark case, the additional loss of monitoring benefits occurs for those signals for which monitoring is more valuable. Consequently, the monitoring effect of financing efficiency becomes more pronounced as well. For some levels of signal quality and private benefits, the monitoring effect becomes even the dominating effect so that the cumulative effect of financing efficiency *and* investment efficiency on firm value turns negative.

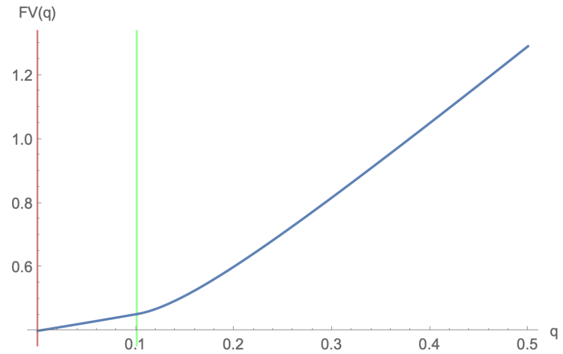
Following this rationale, there exist three levels of private benefits resulting in different effects on firm value. For low levels of private benefits, $0 < B < \underline{B}$, the additional loss of monitoring benefits due to private benefits is so small that for all levels of signal quality the overall effect of investment efficiency and financing efficiency on firm value remains weakly positive. Figure 3a illustrates firm value for low private benefits where the green line indicates the onset of investment efficiency and the red line the onset of financing efficiency. However, for intermediate private benefits, $\underline{B} < B < \overline{B}$, the loss of monitoring benefits may become the dominating effect. Particularly, this is the case for signals that are rather opaque as the private lender's liquidation option is more valuable in these situations. As the signal becomes less opaque, the value of monitoring decreases for good signals so that the loss of monitoring benefits due to private benefits becomes less severe. In this case, the overall effect of signal quality on firm value turns positive resulting in a U-shaped function of firm value as depicted in figure 3b. Lastly, for high private benefits, $\overline{B} < B$, the manager enters the public market whenever possible. The loss of monitoring benefits due to private benefits reaches its maximum so that the firm value is strictly lower than in any of the other cases. Yet, increasing signal quality is still beneficial because it moderates the loss of monitoring benefits. Put differently, given that the monitoring benefits are lost anyway one can at least reduce the value of the benefits by increasing signal quality. Thus, for high private benefits, firm value is strictly increasing in signal quality (figure 3c).



(a) Low private benefits $0 < B < \underline{B} < \overline{B}$.



(b) Intermediate private benefits $0 < \underline{B} < B < \overline{B}$.



(c) High private benefits $0 < \underline{B} < \overline{B} < B$.

Figure 3: Expected firm value as a function of signal quality with different levels of private benefits. Parameter values: $K = 10$, $\alpha = 0.8$, $g = 0.01$, and $c_m = 8$. The green line indicates q_L and the red line $q_I(B)$.

3.4 Optimal signal quality

We have established that private benefits affect managers' debt financing choices. Importantly, this effect is not driven by a pure signal cost argument, highlighting the informational role of the signal. We next turn to the optimal signal quality and potential policy implications. Under the simplifying assumption of costless reporting, the optimal signal quality is the maximum possible quality, i.e., $q^* = 1$, for any level of private benefits. While we assume costless disclosure for exposition, it creates an imbalance because information gathering and monitoring for lenders remains costly, whereas the signal generating process is not. Once this imbalance is resolved by either information gathering becoming costly or by monitoring becoming costless, i.e., $c_m = 0$, the maximum signal quality may not be optimal.

To exemplify the effect of disclosure cost, suppose a standard convex cost function. For low costs, results are qualitatively the same as in the no disclosure cost case. As such, the firm value increases in the signal quality up to the optimum level. However, the optimum shifts from $q^* = 1$ to an interior solution. For moderate costs, increasing signal quality is only optimal for the manager in case of low or high private benefits, respectively. For intermediate private benefits, reducing signal quality can, in fact, increase firm value. Recall that private benefits distort the manager's incentives to enter the public market, and the manager enters the public market too often. The resulting loss of monitoring benefits is particularly high when the signal is opaque because monitoring benefits are more valuable for low signal qualities. When information gathering costs impede a high signal quality, it becomes more valuable not to enter the public market at all and save the costs of information gathering. Therefore, the optimal signal quality for intermediate private benefits is $q^* = 0$. For high costs, information gathering is never optimal because the costs from information gathering exceed all benefits from entering the public market for any level of private benefits.

Alternatively, consider rebalancing the model by setting the costs of monitoring to zero. In this case, private lenders always choose $m^* = 1$ as the optimal monitoring effort. Compared to the main analysis, this results in higher monitoring benefits for good signals. However, we observe a higher loss of monitoring benefits with private benefits compared to the main analysis without private benefits. The result is a situation similar to the case with moderate information gathering costs where it may be optimal to reduce signal quality to maximize firm value. Note that this result does not critically depend on assuming $m^* = 1$ but can be derived for any other fixed level of monitoring effort, e.g., if a standard-setter mandates a minimum level of monitoring effort. There will be a threshold level of monitoring for which the no information, $q = 0$, dominates full information, $q = 1$.

4 Conclusion

We study the effects of private benefits of control on the optimal quality of mandatory disclosure in a debt financing setting. We show that increasing the quality of disclosures can have positive or negative effects on social welfare. The positive effects arise from lenders' ability to more efficiently allocate capital and constrain funding of value-destroying projects. The negative effect is an unattended consequence of allocating debt more efficiently. Public lenders that cannot overcome information asymmetries after initiation react stronger to reliable good news, allowing managers to enter the public market and protect their private benefits of control even if utilizing the monitoring abilities of private lenders is socially desirable.

We find that the positive effects outweigh their unintended consequences when private benefits are weak or strong but not for medium levels. If private benefits are weak, managers predominantly focus on firm value and are unlikely to choose inefficient debt financing for the purpose of staying in control. If private benefits are strong, managers always prefer public debt, making a higher disclosure quality desirable to discipline managers' financing choices for bad news. It cannot change the behaviour for good news. But, for medium levels of private benefits, higher quality disclosures can allow managers to act on their proclivity to avoid private lenders' monitoring. The public debt market reacts stronger to good news, which reduces the manager's cost of protecting private benefits.

In sum, our paper highlights that firms' financing choices and, more importantly, the optimal disclosure quality depends on managers' private benefits. Requiring a high disclosure quality may not be socially desirable in such a setting.

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Appendix

For the sake of readability we define: $E[p \mid q, s] = \pi(q)$

Proof of Lemma 1 The lenders' expected payoffs are given by (2) and (3), respectively. Solving (2) for D_P yields the minimum face value in the public market

$$D_P\left(s, q\right) \geq \frac{1}{\pi(q)} K = \underline{\mathrm{D}}_P \tag{17}$$

The first order condition (FOC) for the private lender calculates as follows

$$(1 - \pi(q))\alpha K - m c_m = 0 \tag{18}$$

Solving the FOC for m yields the optimal monitoring effort

$$m^* = \frac{(1 - \pi(q))\alpha K}{c_m}. \tag{19}$$

Substituting m^* back into (3) and solving for D_L yields the minimum face value in the private market

$$D_L\left(m^*, s, q\right) \geq \frac{1}{\pi(q)} \left(K + gK - \frac{(1 - \pi(q))^2 \alpha^2 K^2}{2 c_m} \right) = \underline{\mathrm{D}}_L \tag{20}$$

The upper limit for D_i results from the fact that the lenders' claim is limited by the available cash flow in the last period $X_S = 2K$. Any $D > X_S$ could never be fully repaid. Note that including a covenant that allows the lender to assume control is ex-ante optimal because the monitoring benefit is positive, reduces the optimal face value, and thus the manager's utility. Absent a covenant, $m^* = 0$, $MB = 0$. □

Proof of Corollary 1 First, note that differentiating equation (1) with respect to q yields

$$\frac{\partial \pi(q)}{\partial q} = s - 1/2 \begin{cases} > 0 & \text{if } s(q) > 1/2 \\ < 0 & \text{if } s(q) < 1/2. \end{cases} \tag{21}$$

Second, we define the risk premium (RP) and the monitoring benefit (MB) component of the face value:

$$RP(q,s) = \frac{1}{\pi(q)} \tag{22}$$

$$MB(q,s) = \frac{(1-\pi(q))^2\alpha^2K^2}{2c_m} \tag{23}$$

We can now write the derivatives for D_i as defined in (20) and (17) as

$$\frac{\partial D_P(q,s)}{\partial\, q} = \underbrace{-\frac{\frac{\partial\pi(q)}{\partial q}}{\pi(q)^2}}_{\partial RP(q,s)/\partial q}\;K \tag{24}$$

and

$$\frac{\partial D_L(q,s)}{\partial\, q} = -\frac{\frac{\partial\pi(q)}{\partial q}}{\pi(q)^2}(K+gK+MB(q,s)) + RP(q,s)\underbrace{\frac{\big(1-\pi(q)\big)\alpha^2K^2}{2c_m}\frac{\partial\pi(q)}{\partial q}}_{\partial MB(q,s)/\partial q} \tag{25}$$

Because $0<\pi(q)<1$ it must hold that $\text{sign}(\partial RP(q,s)/\partial q) = -\text{sign}(\partial MB(q,s)/\partial q) = -\text{sign}(\partial\pi(q)/\partial q)$. Consequently, (24) must be strictly negative. To determine the sign of (25) we must evaluate which of the two effects dominates. Collecting terms and rearranging yields

$$\begin{aligned}\frac{\partial\, D_L(q,s)}{\partial\, q} &= -\frac{\frac{\partial\pi(q)}{\partial q}}{\pi(q)^2}(K+gK) + \left(-\frac{MB(q,s)}{\pi(q)^2} + RP(q,s)\frac{\big(1-\pi(q)\big)\alpha^2K^2}{c_m}\right)\frac{\partial\pi(q)}{\partial q}\\ &= \quad -\frac{\frac{\partial\pi(q)}{\partial q}}{\pi(q)^2}(K+gK) + \frac{\big(1-\pi(q)^2\big)\alpha^2K^2}{2c_m\pi(q)^2}\frac{\partial\pi(q)}{\partial q}\end{aligned} \tag{26}$$

In the next step, we show that the former effect dominates. Therefore, we have to distinguish two cases because $\frac{\partial\pi(q)}{\partial q}$ is ambiguous. Assume first, that $\frac{\partial\pi(q)}{\partial q}>0$, which is the case for $s>1/2$, and that the latter term dominates, i.e. (26) >0 Simplifying the inequality yields

$$g < \frac{(1-\pi(q)^2)\alpha^2K}{2c_m} - 1. \tag{27}$$

Consider the boundaries on g, $0 < g < \frac{\alpha^2 K}{8c_m} < 1$. The upper bound is obviously uncritical, whereas we have to check on the lower bound. Imposing the lower bound and further simplifying yields

$$c_m < (1 - \pi(q)^2)\alpha^2 K/2 \tag{28}$$

which yields contradiction because $c_m > \alpha K$, $0 < \alpha < 1$ and $(1 - \pi(q)^2) < 1$. Consequently, the former effect dominates (26) if $\frac{\partial \pi(q)}{\partial q} > 0$.

Second, assume that $\frac{\partial \pi(q)}{\partial q} < 0$, which is the case for $s < 1/2$, and that the latter term dominates, i.e. (26) < 0. Simplifying the inequality again yields

$$g < \frac{(1 - \pi(q)^2)\alpha^2 K}{2c_m} - 1. \tag{29}$$

yielding contradiction as we have shown above. Again, the former effect dominates (26) if $\frac{\partial \pi(q)}{\partial q} < 0$. $\square$

Proof of Lemma 2 Recall, that lenders only finance the project if $D_i \leq 2K$ as defined in (7) and (8). Solving these inequalities yields the thresholds for which the manager can obtain financing in the respective markets. In the private market the lender accepts financing if

$$s_L(q) = \frac{1+q}{2q} - \frac{4c_m - \Omega}{2q\alpha^2 K} \leq s \tag{30}$$

where $\Omega = \sqrt{8c_m\left(2c_m - (1-g)\alpha^2 K\right)}$ and in the public market the lender accepts financing if $s_P = 1/2 \leq s$, respectively. Further, we have to show that $s_L(q) < 1/2 = s_P$. Assume for the contrary, that $s_L(q) > 1/2$. Rearranging for g yields $g > \alpha^2 K/8c_m$ which contradicts the assumption on g. Lastly, we can derive the threshold on q for which $s_L(q) \geq 0$. Solving for q yields

$$q_L = \frac{4c_m - \Omega}{\alpha^2 K} - 1 \tag{31}$$

$\square$

Proof of Proposition 1 In equilibrium the manager's conjecture about the private lender's monitoring effort is correct, so that $\hat{m} = m^*$. First note, that if the manager obtains financing in one the debt markets, he maximizes his expected payoff by offering lenders the minimum face value, $D_L = \underline{D}_L$ and $D_P = \underline{D}_P$, respectively. The reason is that because of the competitive market assumption lenders are willing to accept a debt contract with the minimum face value and any face value above the minimum face value would reduce the manager's expected payoff. Further note, that because the manager is protected by limited liability, he always wants to conduct the project if financing is obtainable.

From Lemma 2 we know that if the signal is sufficiently precise, i.e. $q_L < q \leq 1$, then $s_L(q) > 0$. For signals $0 \leq s < s_L(q)$ the debt market does not accept the debt contract and therefore the manager obtains no financing. Reversely, for any signal $s_L(q) \leq s \leq 1$ the manager obtains financing. Combining these findings yields the first interval (a) of the Proposition.

From Lemma 2 further follows that the manager can choose between the private and public market only for signals $s_P \leq s \leq 1$. Consider first the interval of signals $s_L(q) \leq s < s_P$. Obtaining financing in the private market yields a non-negative expected payoff whereas no financing yields a payoff of zero. Hence, the manager always obtains financing in the private market for this interval by offering face value $D_L = \underline{D}_L$. In the interval $s_P \leq s \leq 1$, the manager maximizes his expected payoff by (i) offering the minimum face value in the respective market and (ii) choosing the debt market that yields a (weakly) higher expected payoff. We can write this trade-off in terms of the manager's preference for the public market as an indifference condition

$$\pi(q)(X_S - D_L) + (1 - \hat{m}(1 - \pi(q)))B \geq \pi(q)(X_S - D_P) + B \tag{32}$$

where the LHS derives from (6) and the RHS from (5). Solving the indifference condition, we find that the manager prefers the public market if

$$s_I(q, B) = 1/2 + 1/(2q)\left(1 - \sqrt{\Phi(B)}\right) \leq s \tag{33}$$

with $\Phi(B) = (8gc_m)/(\alpha(\alpha K - 2B))$. Solving $s_I(q, B) = 1$ for q yields $q_I(B) = 1 - \sqrt{\Phi(B)}$.

Calculating the derivative of $s_I(q,B)$ from (33) with respect to B yields

$$\frac{\partial s_I(q,B)}{\partial B} = -\frac{\sqrt{\Phi(B)}}{2q(\alpha K - 2B)}. \tag{34}$$

Note here, that the derivative is negative if $\alpha K > 2B$, and therefore, the manager's indifference threshold $s_I(q,B)$ may fall below $s_P = 1/2$ for large B. If this is the case, the lender's financing constraint becomes binding and the manager's preference becomes irrelevant. Solving $s_I(q,B) = s_P = 1/2$ for B yields the threshold

$$\overline{B} = \frac{\alpha K}{2} - \frac{4gc_m}{\alpha} < \frac{\alpha K}{2}. \tag{35}$$

Hence, we can conclude that on the relevant domain, $1/2 \leq s_I(q,B) \leq 1$, the manager's indifference threshold $s_I(q,B)$ is strictly decreasing in B because here it holds that $\alpha K > 2B$. Moreover, we can conclude that the manager can only obtain financing in the public market for signals $\max(s_P, s_I(q,B)) \leq s \leq 1$.

Summing up the above findings, in equilibrium the manager obtains financing in the private market for the interval $s_L(q) \leq s < \max(s_P, s_I(q,B))$ by offering face value $D_L = \underline{D}_L$ (interval (b)) and financing in the public market for the interval $\max(s_P, s_I(q,B)) \leq s \leq 1$ by offering face value $D_P = \underline{D}_P$ (interval (c)). $\square$

Proof of Corollary 3 Rearranging the derivative of $s_L(q)$ as defined in (30) with respect to q yields

$$\frac{\partial\, s_L(q)}{\partial\, q} = \frac{1}{q}\left(\frac{1}{2} - s_L(q)\right) \tag{36}$$

which by Lemma 2 must be strictly positive.

Likewise, we can perform a similar operation for the derivative of $s_I(q,B)$ as defined in (33) with respect to q yielding

$$\frac{\partial\, s_I(q,B)}{\partial\, q} = \frac{1}{q}\left(\frac{1}{2} - s_I(q,B)\right) \tag{37}$$

which by Proposition 1 must be strictly negative. $\square$

Proof of Proposition 2 Since we assume $B = 0$, we omit B in this proof for notational ease.

At $q = 0$, $s_L(q) < 0$ and $s_I(q) > 1$. From the proof of Lemma 3 and Proposition 1 we know q_L and q_I. Considering that $s_L(q) < 1/2 \leq s_I(q)$, we can calculate the expected firm value (hereafter only 'firm value') for the intervals $q \in [0, q_L), q \in [q_L, q_I)$ and $q \in (q_I, 1]$. For each of the intervals we prove that firm value is increasing in q.

We define the firm value given signal s in each market as follows:

$$V(Pu \mid q, s) = \pi(q)(X_S - D_P) = \pi(q)2K - K \tag{38}$$

$$V(Pr \mid m^*, q, s) = \pi(q)(X_S - D_L) = \pi(q)2K - K + \frac{(1 - \pi(q))^2 \alpha^2 K^2}{2c_m} - gK \tag{39}$$

(i) Firm value for $q \in [0, q_L)$

$$\begin{aligned} V(0 \leq q < q_L) = \int_0^1 V(Pr \mid m^*, q, s) ds &= E[\pi(q)]2K - K + \frac{\alpha^2 K^2 (3 + q^2)}{24c_m} - gK \\ &= \frac{\alpha^2 K^2 (3 + q^2)}{24c_m} - gK \end{aligned} \tag{40}$$

Note that $E[\pi(q)] = 1/2$ so that the ex-ante NPV absent financing effects is zero. The remaining firm value consists of the monitoring benefit net of capital rationing costs. One can easily see from (40) that firm value is increasing in q because a more precise signal enables more efficient monitoring by the lender.

(ii) Firm value for $q \in (q_L, q_I]$

$$\begin{aligned} V(q_L < q \leq q_I) &= \int_{s_L(q)}^1 V(Pr \mid m^*, q, s)\, ds \\ &= (40) + \underbrace{\int_0^{s_L(q)} \left[-\underbrace{\left(\pi(q)2K - K\right)}_{PR} - \underbrace{\left(\frac{(1 - \pi(q))\alpha^2 K^2}{2c_m}\right)}_{MB} + \underbrace{\left(gK\right)}_{CRC} \right] ds}_{investment\ efficiency} \end{aligned} \tag{41}$$

Calculating the total derivative with respect to q yields

$$\begin{aligned}\frac{dV(q_L < q \leq q_I)}{dq} = \frac{\partial(40)}{\partial q} + PR(s_L(q))\frac{\partial}{\partial q}s_L(q) + \int_0^{s_L(q)} \frac{\partial}{\partial q}PRds \\ + MB(s_L(q))\frac{\partial}{\partial q}s_L(q) + \int_0^{s_L(q)} \frac{\partial}{\partial q}MBds + gK\frac{\partial}{\partial q}s_L(q)\end{aligned} \tag{42}$$

Collecting terms yields

$$\begin{aligned}\frac{dV(q_L < q \leq q_I)}{dq} = \frac{\partial(40)}{\partial q} + \Big(PR(s_L(q)) + MB(s_L(q)) + gK\Big)\frac{\partial}{\partial q}s_L(q) \\ + \int_0^{s_L(q)} \left(\frac{\partial}{\partial q}PR + \frac{\partial}{\partial q}MB\right) ds\end{aligned} \tag{43}$$

Note first, that we have already shown that (40) is increasing in q. Hence, if we can show that the effect of signal quality on investment efficiency is positive the overall effect must be positive as well. We label the second term of the RHS as the indirect effect of signal quality on investment efficiency and the third term of the RHS as direct effect of signal quality on investment efficiency.

We can rewrite the indirect effect as

$$\left(-(E[p \mid s_L(q)]2K - K) - \frac{(1 - E[p \mid s_L(q)])^2\alpha^2K^2}{2c_m} + gK\right) * \frac{\partial}{\partial q}s_L(q) \tag{44}$$

Note here, that the indirect effect is evaluated at the point $s_L(q)$ which is defined via the financing constraint of the private lender. The signal indicates the point where where the effects exactly balance, i.e. where the marginal effect is exactly zero. Therefore the term in brackets collapses to zero so that we are left with the direct effect of signal quality on investment efficiency.

Computing the derivatives for the direct effect yields

$$\int_0^{s_L(q)} \left(\frac{\partial}{\partial q} PR + \frac{\partial}{\partial q} MB\right) ds$$

$$= \int_0^{s_L(q)} \frac{\partial}{\partial q} \left[-\left(\pi(q)2K - K\right) - \left(\frac{(1-\pi(q))^2 \alpha^2 K^2}{2c_m}\right)\right] ds \tag{45}$$

$$= (1 - s_L(q)) s_L(q) K + \frac{\alpha^2 K^2 s_L(q)}{12 c_m}(3 - 3s_L(q) + q\left(3 + 4s_L(q)^2 - 6s_L(q)\right))$$

We prove that (45) is positive by contradiction; assume that (45) < 0 instead. For some $0 < q < 1$, $s_L(q)$ will reach its lower bound of $s_L(q) = 0$. In this case, the direct effect amounts to

$$\frac{\partial IE}{\partial q}|_{s_L(q)=0} = 0 \tag{46}$$

yielding contradiction. From Corollary (3) we know that $s_L(q)$ is strictly increasing q and is limited by $s_L(q) = 1/2$. We can use this property to show that (45) is monotone and (weakly) increasing in q by showing that (45) is monotone and (weakly) increasing in $s_L(q)$. Suppose that IE is decreasing in $s_L(q)$:

$$\frac{\partial IE}{\partial s_L(q)} = \frac{K(1 - 2s_L(q))\left(4c_m - \alpha^2 K(1 + q(1 - 2s_L(q)))\right)}{4c_m} < 0. \tag{47}$$

Rearranging yields

$$c_m < \alpha^2 K(1 + q(1 - 2s_L(q)))/4$$

which yields contradiction because $\alpha K < c_m, \alpha < 1$ and $s_L(q) < 1/2$. Consequently, the derivative must be (weakly) positive implying that signal quality has a positive effect on investment efficiency and thus firm value.

(iii) Firm value for $q \in (q_I, 1]$

$$V(q_I < q \le 1) = \int_{s_L(q)}^{s_I(q)} V(Pr \mid m^*, q, s)\, ds + \int_{s_I(q)}^{1} V(Pu \mid q, s) ds$$

$$= (41) + \underbrace{\int_{s_I(q)}^{1} \left[\underbrace{\left(-\frac{(1-\pi(q))^2\alpha^2 K^2}{2c_m}\right)}_{MB} + \underbrace{\left(gK\right)}_{CRC}\right] ds}_{\textit{financing efficiency}} \tag{48}$$

Again, we calculate the total derivative with respect to q

$$\frac{dV(q_I < q \leq 1)}{dq} = \frac{\partial(41)}{\partial q} + \Big(MB(s_I(q)) + gK\Big)\frac{\partial}{\partial q}s_I(q) + \int_{s_I(q)}^{1}\left(\frac{\partial}{\partial q}MB\right) ds \tag{49}$$

Note first, that we have already shown that (41) is increasing in q. Hence, if we can show that the effect of signal quality on financing efficiency is positive the overall effect must be positive as well. We label the second term of the RHS as the indirect effect of signal quality on financing efficiency and the third term of the RHS as direct effect of signal quality on financing efficiency.

Note further, that the indirect effect is evaluated at the point $s_I(q)$ which is defined via the manager's indifference condition. The signal indicates the point where where manager is indifferent between the public and the private market. Put differently, where the effects from both markets exactly balance. Therefore the term in brackets collapses to zero so that we are left with the direct effect of signal quality on financing efficiency.

Computing the derivative for the direct effect yields

$$\int_{s_I(q)}^{1}\frac{\partial}{\partial q}\left[-\left(\frac{(1-\pi(q))^2\alpha^2K^2}{2c_m}\right)\right] ds = -\frac{\alpha^2k^2(1-s_I(q))\left(q(1-2s_I(q))^2-(3-2q)s_I(q)\right)}{12c_m} \tag{50}$$

which is strictly positive because $s_I(q) > 1/2$. Consequently, signal quality has a positive effect on financing efficiency and thus firm value. □

Proof of Lemma 3 The derivative of $s_I(q,B)$ with respect to B as well as the calculation of the threshold $\overline{B}$ is included in the proof of Proposition 1. □

Proof of Proposition 3 The line or argumentation is as follows:

I. For $0 \leq q < q_I(B)$ firm value is strictly increasing in q for all $B > 0$. Hence, we can restrict attention to the case $q_I(B) \leq q \leq 1$.

II. For $q_I(B) \leq q \leq 1$ firm value is defined by

   (a) equation (51) given $0 < B < B'$ where B' satisfies $q_I(B) = q_L$ such that $0 < q_L \leq q_I(B) < 1$, and

(b) equation (59) given $B' \leq B < \overline{B} \wedge q_I(B) \leq q \leq q_L$ and equation (51) given $B' \leq B < \overline{B} \wedge q_L < q \leq 1$.

We show that for $q \leq 0$ equation (51) and equation (59) are U-shaped in q with a unique minimum denoted $\underline{q}$ and $\overline{q}$, respectively.

III. We show that there is one unique minimum that lies in the interval $q_I(B) \leq q \leq 1$ yielding case (ii) of the Proposition or that the respective minimum lies below the interval $q_I(B) \leq q \leq 1$ yielding cases (i) and (iii) of the Proposition. Now, considering II., firm value is U-shaped in q in the interval $q_I(B) \leq q \leq 1$ for case (ii) and strictly increasing in q in the interval $q_I(B) \leq q \leq 1$ for cases (i) and (iii).

IV. We show that for $B \geq \overline{B}$ firm value is strictly increasing in q.

Proof of I.

Private benefits only affect the financing efficiency whereas the base value and investment efficiency remain unaffected. In terms of firm value, the effect of private benefits is therefore limited to the interval $q_I(B) \leq q \leq 1$. In the interval $0 \leq q < q(B)$ firm value is strictly increasing in q which follows from Proposition 2.

Proof of II.

The proof of II. consists of two steps. We show that

II.1 equation (51) has a single positive root $\underline{q}(B)$ and

II.2 equation (59) has a single positive root $\overline{q}(B)$.

Proof of II.1: (51) has a single positive root $\underline{q}(B)$

In the interval $q_L < q_I(B) \leq q < q_L$ firm value computes to:

$$
\begin{aligned}
&V(0 < B < B', q_I(B) < q \leq 1) \\
&= \int_{s_L(q)}^{s_I(q,B)} V(Pr \mid s, m^*, B) ds + \int_{s_I(q,B)}^{1} V(Pu \mid s, B) ds \qquad (51) \\
&= \frac{\Gamma + \Delta(B) + 3K\left(q^2 + \Lambda(B) - 1\right)}{12q}
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma &= \frac{4(1-g)(6c_m - \Omega)}{\alpha^2} - \frac{8c_m(4c_m - \Omega)}{\alpha^4 K} \\
\Delta(B) &= -\frac{K^2\sqrt{32\alpha c_m g^3}}{(\alpha K - 2B)^{3/2}} \\
\Lambda(B) &= \sqrt{\frac{32 c_m g^3}{\alpha(\alpha K - 2B)}}.
\end{aligned}
$$

Calculating the first order condition (FOC) and rearranging yields

$$
q^2 - \left(\frac{\Gamma + \Delta(B)}{3K} + \Lambda(B) - 1\right) = 0 \qquad (52)
$$

By Descartes' rule of changes in signs, the FOC has a single positive root if $\Gamma + \Delta(B) + 3K\Lambda(B) - 3K > 0$. Consider the derivative of $\Gamma + \Delta(B) + 3K\Lambda(B) - 3K$ with respect to B:

$$
\frac{\partial(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)}{\partial B} = -\frac{BK\sqrt{8c_m g^3}}{\sqrt{\alpha}(\alpha K - 2B)^{5/2}} \qquad (53)
$$

which is strictly negative in this case because $B < B' < \alpha K/2$. Hence, showing that $(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B}} > 0$ implies $\Gamma + \Delta(B) + 3K\Lambda(B) - 3K > 0 \, \forall B \in [0, B']$ because $B' < \overline{B}$. Assume for the contrary that $(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B}} < 0$. For $\alpha = 0$ we get $(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B},\alpha=0} = 0$ which yields contradiction.

For $\alpha = 1$ we get

$$
\begin{aligned}
(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B},\alpha=1} &= \frac{64c_m^{5/2}\sqrt{4c_m - 2(1-g)K} + 96c_m^2(1-g)K}{48c_m K} \\
&- \frac{\left(128c_m^3 + 4c_m K\left(8(1-g)\sqrt{2c_m(2c_m + (g-1)K)} + 3(1-2g)K\right) + K^3\right)}{48c_m K}
\end{aligned}
\tag{54}
$$

Assume that this expression is negative.

The derivative of (54) with respect to g calculates as

$$
\frac{\partial(54)}{\partial g} = K/2 - 2c_m + \sqrt{2c_m(2c_m(1-g)K)} < 0 \tag{55}
$$

which is negative because for $\alpha = 1$, $0 < g < K/8c_m$. Since (54) is decreasing in g it is sufficient to show that the function is non-negative at the limit $g = K/8c_m$.

For $g = K/8c_m$ we get,

$$
(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B},\alpha=1,g=K/8c_m} = 0. \tag{56}
$$

Hence, we can conclude that $(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B},\alpha=1} > 0$.

What is left to show is that $(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B}}$ is monotone in α. Assume that the derivative with respect to α is negative

$$
\frac{\partial(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B}}}{\partial\alpha} < 0 \tag{57}
$$

. Simplifying yields $g > \frac{\alpha^2 K}{8c_m}$ contradicting the assumption on g. Hence, $(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B}}$ is monotonously increasing in α and $(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)|_{B=\overline{B}} > 0$. Consequently, the function must have a single positive root which computes to

$$
\underline{q}(B) = \sqrt{\frac{\Gamma + \Delta(B)}{3K} + \Lambda(B) - 1}\,. \tag{58}
$$

Proof of II.2: (59) has a single positive root $\overline{q}(B)$

The proof differs from the proof of II.1 in that in the interval $q_I(B) < q \leq q_L$ only FE occurs but not IE. The firm value calculates as

$$
\begin{aligned}
&V(B' < B < \overline{B}, q_I(B) < q \leq q_L) \\
&= \int_0^{s_I(q,B)} V(Pr \mid s, m^*, B)ds + \int_{s_I(q,B)}^1 V(Pu \mid s, B)ds \\
&= \frac{K\left(\alpha(q+1)(\alpha K - 2B)^2\left(\alpha^2 K(q+1)^2 - 24c_m g\right) + \Theta\right)}{48\alpha c_m q(\alpha K - 2B)^2}
\end{aligned}
\tag{59}
$$

where $\Theta = 16(\alpha K - 3B)c_m g\sqrt{8\alpha c_m g(\alpha K - 2B)}$. Taking the FOC and rearranging yields

$$
2\alpha^2 K q^3 + 3\alpha^2 K q^2 + \left(\left(24c_m g - \alpha^2 K\right) - \frac{\Theta}{\alpha(\alpha K - 2B)^2}\right) = 0 \tag{60}
$$

By Descartes' rule of changes in signs, the derivative has a single positive root if

$$
\left(\left(24c_m g - \alpha^2 K\right) - \frac{\Theta}{\alpha(\alpha K - 2B)^2}\right) < 0 \tag{61}
$$

Assume for the contrary that (61) > 0. Solving for B then yields

$$
B > \frac{\alpha K}{2} - \frac{4c_m g}{\alpha} = \overline{B} \tag{62}
$$

Hence, for any $B \in (0, \overline{B}]$ the function has a single positive root $\overline{q}(B)$ which is implicitly defined in (60).

Proof of III.

The proof of III. consists of five steps.

Consider first the case where $0 < B < B'$. We show that

III.1 at the limits $B = 0$ and $B = B'$ the derivative with respect to q is $\frac{d}{dq}V(B = 0)|_{q=q_I(B)} > 0$ and $\frac{d}{dq}V(B = B')|_{q=q_I(B)} < 0$, respectively.

From this follows that at $B = 0$, $\underline{q} < q_I(B)$ and at $B = B'$, $q_I(B) < \underline{q}$. Hence, there must exist at least one switching point $\underline{B}$ in the interval $0 < B < B'$ below which $\underline{q}(B) < q_I(B)$ (case *i.* of the Proposition) and above which $q_I(B) < \underline{q}(B)$ (case *ii.* of the Proposition).

To prove uniqueness of the switching point, we show that

III.2 the difference between $q_I(B)$ and $\underline{q}(B)$ is decreasing and monotone, i.e. $\dfrac{d(q_I(B) - \underline{q}(B))}{dB} < 0$

Finally, to ensure that we are not only on the decreasing part of the function, we show that

III.3 firm value is strictly larger at $q = 1$ compared to $q = q_I(B)$, i.e. $V(q = q_I(B)) < V(q = 1) \quad \forall B$

implying that $\underline{q}(B) < 1$ and that firm value must be U - shaped in q for $\underline{B} < B \leq B'$.

Consider next the case where $B' < B < \overline{B}$. We show that

III.4 firm value is always decreasing at $q = q_I(B)$, i.e. $\dfrac{d}{dq}V(B' < B < \overline{B})|_{q=q_I(B)} < 0$ and

III.5 the derivative of both firm value functions with respect to q are the same at $q = q_L$, i.e. $\dfrac{dV(q_I(B) \leq q < q_L)}{dq}|_{q=q_L} = \dfrac{dV(q_L \leq q \leq 1)}{dq}|_{q=q_L}$, which implies that the firm value function in the interval $q_I(B) < q \leq 1$ is smooth.

These properties together with the functional form of both functions (strictly convex with a unique minimum), proves that there exists only *one* minimum in $q_I(B) < q \leq 1$. We have already shown that $V(q = q_I(B)) < V(q = 1) \quad \forall B$. Consequently, the minimum lies within the interval $q_I(B) < q < 1$ which means that firm value must be U - shaped in q for $B' < B < \overline{B}$ (completing the proof of case *ii.* of the Proposition).

Proof of III.1: $\dfrac{d}{dq}V(B = 0)|_{q=q_I(B)} > 0$ *and* $\dfrac{d}{dq}V(B = B')|_{q=q_I(B)} < 0$

We can decompose the firm value into three components: the base firm value (BV) as calculated in (40), investment efficiency (IE) as calculated in (41) and financing efficiency (FE) as calculated in (16). In the proof of Proposition 2, we have already shown that for $B = 0$

$$\frac{d}{dq}BV > 0, \quad \frac{d}{dq}IE > 0 \quad \text{and} \quad \frac{d}{dq}FE > 0 \quad \forall q \tag{63}$$

which implies $\dfrac{d}{dq}V(B = 0)|_{q=q_I(B)} > 0$.

Moreover, we know that private benefits only affect FE while BV and IE remain unchanged. Thus, the derivative of firm value turns negative iff

$$\frac{d}{dq}BV|_{q=q_I(B)} + \frac{d}{dq}IE|_{q=q_I(B)} < -\frac{d}{dq}FE|_{q=q_I(B)}. \tag{64}$$

Assume for the contrary that

$$\frac{d}{dq}BV|_{q=q_I(B)} + \frac{d}{dq}IE|_{q=q_I(B)} \geq -\frac{d}{dq}FE|_{q=q_I(B)} \tag{65}$$

Making use of the fact that $s_I(q_I(B)) = 1$, we get

$$\begin{aligned} &K(1-s_L(q_I(B)))s_L(q_I(B)) + \frac{\alpha^2K^2q_I(B)}{12c_m} + \left(\frac{1}{2q_I(B)} - \frac{s_L(q_I(B))}{q_I(B)}\right) * \\ &\quad \left(\frac{\alpha^2K^2(1-q_I(B)(1-2s_L(q_I(B))))^2}{8c_m} + K(g+q_I(B)(1-2s_L(q_I(B))))\right) \\ &\quad - \frac{\alpha^2K^2s_L(q_I(B))\left(q_I(B)\left(4s_L(q_I(B))^2-6s_L(q_I(B))+3\right)+3(1-s_L(q_I(B)))\right)}{12c_m} \\ &\quad > -\left(-\frac{BgK}{q_I(B)(\alpha K-2B)}\right) \end{aligned} \tag{66}$$

Recall that for $B = 0$, $q_L < q_I(B)$. Now, since q_L is a constant and $q_I(B)$ is decreasing in B, there must exist a $B = B'$ for which $q_I(B) = q_L$. Solving $q_I(B) = q_L$ for B yields

$$B' = \frac{\alpha K}{2} - \frac{4c_mg}{\alpha(1-q_L)^2} < \overline{B} \tag{67}$$

where the inequality follows from $0 < q_L < 1/2$. Using the fact that $s_L(q_L) = 0$, plugging B' into (66) and simplifying yields

$$\frac{K\left(24c_m(g+q_L)+\alpha^2K\left(q_L^2-6q_L-3\right)\right)}{48c_mq_L} > \frac{K\left(\alpha^2K(1-q_L)^2-8c_mg\right)}{16c_mq_L} \tag{68}$$

Solving for q_L and considering that $c_m > \alpha K$ and $0 < \alpha < 1$, we can establish the following condition for inequality (68) to hold:

$$\frac{6c_m}{\alpha^2K} - \sqrt{3\left(\frac{12c_m^2}{\alpha^4K^2} + \frac{8c_mg}{\alpha^2K} - 1\right)} < q_L < 1 < \frac{6c_m}{\alpha^2K} + \sqrt{3\left(\frac{12c_m^2}{\alpha^4K^2} + \frac{8c_mg}{\alpha^2K} - 1\right)}. \quad (69)$$

Substituting q_L from our previous analysis yields

$$\frac{6c_m}{\alpha^2K} - \sqrt{3\left(\frac{12c_m^2}{\alpha^4K^2} + \frac{8c_mg}{\alpha^2K} - 1\right)} < q_L = \frac{4c_m - A}{\alpha^2K} - 1. \quad (70)$$

Simplifying terms yields

$$2c_m - \sqrt{36c_m^2 + 24\alpha^2c_mgK - 3\alpha^4K^2} < -\alpha^2K - \sqrt{8c_m\left(2c_m - \alpha^2(1-g)K\right)} \quad (71)$$

For $\alpha = 0$ we get $c_m < c_m$ yielding contradiction.
For $\alpha = 1$ we get

$$2c_m - \sqrt{36c_m^2 + 24c_mgK - 3K^2} < -\sqrt{8c_m(2c_m + (g-1)K)} - K \quad (72)$$

Solving for g yields $g > K/(8c_m)$ which contradicts the assumption on g. What is left to show is that both sides are monotone in α. Rearranging $\frac{\partial LHS}{\partial \alpha} > 0$ for g we get $g > \alpha^2K/(4c_m)$ contradicting the assumption on g. Rearranging $\frac{\partial RHS}{\partial \alpha} > 0$ for g we get

$$g_{1,2} = \left(1 + \frac{\alpha^2K}{4c_m} \pm \frac{\sqrt{16c_m^2 + \alpha^4K^2}}{4c_m}\right)$$

It is straight forward to see that the case with the all positive solution violates the assumption on g. Comparing the other solution to $g < \alpha^2K/8c_m$ and simplifying yields $K < 0$ which again yields contradiction. Hence, we can conclude that $\frac{d}{dq}V(B = B')|_{q=q_I(B)} < 0$.

Proof of III.2: $\frac{d(q_I(B) - \underline{q}(B))}{dB} < 0$

Consider first how $q_I(B)$ and $\underline{q}(B)$ change in B. We have already shown that $\partial q_I(B)/\partial B <$

0. Regarding $\underline{q}(B)$ we know that

$$\operatorname{sign}\left(\frac{\partial \underline{q}(B))}{\partial B}\right) = \operatorname{sign}\left(\frac{\partial(\Gamma + \Delta(B) + 3K\Lambda(B) - 3K)}{\partial B}\right). \tag{73}$$

We have already shown for the latter that the derivative is negative, thus $\underline{q}(B)$ must be decreasing in B as well, yet, at a different rate than $q_I(B)$. To ensure uniqueness, we proof that the difference $\Delta q = (q_I(B) - \underline{q}(B))$ is monotonously decreasing in B. We can rewrite $q_I(B)$ in terms of $\Lambda(B)$ as

$$q_I(B) = 1 - \frac{8gc_m}{\alpha(\alpha K - 2B)} = 1 - \sqrt{\frac{32c_m g^3}{4g^2\alpha(\alpha K - 2B)}} = 1 - \frac{\Lambda(B)}{2g}. \tag{74}$$

Now we can write the derivative of Δq with respect to B as

$$\frac{\partial \Delta q}{\partial B} = -\frac{1}{2g}\frac{\partial \Lambda(B)}{\partial B} - \frac{\frac{\frac{\partial \Delta(B)}{\partial B}}{3K} + \frac{\partial \Lambda(B)}{\partial B}}{2\underline{q}(B)} \tag{75}$$

Assume for the contrary that (75) > 0. Rearranging then yields

$$\left(\frac{\underline{q}(B)}{g} + 1\right) > \frac{\Delta(B)}{\Lambda(B)} = -\frac{3\alpha K^2}{\alpha K - 2B}. \tag{76}$$

Rearranging for $\underline{q}(B)$ yields

$$\underline{q}(B) < g\left(\frac{\alpha K}{\alpha K - 2B} - 1\right). \tag{77}$$

Now we know that in this setting it must hold that $q_L < \underline{q}(B)$, so we can write instead

$$q_L = \frac{4c_m - \sqrt{8c_m\left(2c_m - (1-g)\alpha^2 K\right)}}{\alpha^2 K} - 1 < g\left(\frac{\alpha K}{\alpha K - 2B} - 1\right). \tag{78}$$

Note here, that only the RHS depends on B, whereas the LHS is constant in B. Setting $B = 0$ we get $q_L < 0$ which yields contradiction. Setting $B = \overline{B} > B'$ we get

$$\frac{4c_m - \sqrt{8c_m\left(2c_m - (1-g)\alpha^2 K\right)}}{\alpha^2 K} - 1 < \frac{\alpha^2 K}{8c_m} - g. \tag{79}$$

Setting $g = 0$ we get

$$\frac{\alpha^2 K}{8c_m} < 1 < \frac{4c_m - \sqrt{8c_m\,(2c_m - \alpha^2 K)}}{\alpha^2 K} - 1 \tag{80}$$

where the ordering derives from the assumptions on c_m and α yielding contradiction. Setting $g = \alpha^2 K/8c_m$ yields $1 < 0$ which also yields contradiction. Further, the LHS and the RHS are strictly decreasing in g, whereby the RHS decreases at a higher rate. Hence, we can conclude that there is a contradiction for (79). Lastly, it is straight forward to see, that the RHS is linearly increasing in B. Consequently, we get a contradiction for (78) implying that Δq must be strictly decreasing in B.

Proof of III.3: $V(q = q_I(B)) < V(q = 1) \quad \forall B$

From the proof of Proposition 2 we know that BV and IE are strictly increasing in $q\,\forall q$. Hence, we know that $BV(q = q_I(B)) < BV(q = 1)$ and $IE(q = q_I(B)) < IE(q = 1)$. In contrast, FE may be positive or negative depending on B. Calculating FE at $q = q_I(B)$ and $q = 1$ we get

$$FE(q = q_I(B)) = 0 \quad \textit{and} \quad FE(q = 1) = gk(1 - s_I(q, B)) - \frac{\alpha^2 K^2 (2(1 - s_I(q, B)))^3}{48c_m}. \tag{81}$$

Recall that $\lim_{B \to \overline{B}} s(B) = 1/2$. Substituting $s_I = 1/2$ into $FE(q = 1)$ yields

$$FE(q = 1, B = \overline{B}) = \frac{gK}{2} - \frac{\alpha^2 K^2}{48c_m} \tag{82}$$

which can be positive or negative depending on parameter values and which is increasing in g. The loss of firm value resulting from FE reaches its maximum for $g \to 0$

$$FE(q = 1, B = \overline{B}, g = 0) = -\frac{\alpha^2 K^2}{48c_m}. \tag{83}$$

Comparing the maximum loss of firm value from FE at $q = 1$ with the gain of firm value from BV at $q = 1$ we get

$$\frac{\left(4 - q_I(B)^2\right)\left(\alpha^2 K^2\right)}{24c_m} > -\left(-\frac{\alpha^2 K^2}{48c_m}\right) \tag{84}$$

because $0 < q_I(B) < 1$. In sum, the gain from BV always exceeds the loss from FE, and in addition, IE strictly increases firm value. Therefore, it follows that $V(q = q_I(B)) < V(q = 1)\,\forall B$.

Proof of III.4: $\frac{d}{dq}V(B' < B < \overline{B})|_{q=q_I(B)} < 0$

For $B' < B < \overline{B}$ the firm value at $q = q_I(B)$ only consists of BV and FE because $q_I(B) < q_L$. Therefore, the derivative of firm value with respect to q at the point $q_I(B)$ turns negative iff

$$\frac{d}{dq}BV|_{q=q_I(B)} < -\frac{d}{dq}FE|_{q=q_I(B)}. \tag{85}$$

Assume for the contrary that

$$\frac{d}{dq}BV|_{q=q_I(B)} \geq -\frac{d}{dq}FE|_{q=q_I(B)}. \tag{86}$$

Simplifying yields

$$\frac{\alpha^2 K q_I(B)}{12c_m} \geq \frac{Bg}{q_I(B)(\alpha K - 2B)}. \tag{87}$$

The RHS is increasing in B. The derivative of the LHS with respect to B is

$$\frac{\partial LHS}{\partial q} = -\frac{\sqrt{2}4c_m g K q_I(B)}{\alpha(\alpha K - 2B)^2\sqrt{\frac{c_m g}{\alpha(\alpha K - 2B)}}} \tag{88}$$

which is negative. Consequently, to show that $\frac{d}{dq}V(B' < B < \overline{B})|_{q=q_I(B)} < 0$ for any $B > B'$, it is sufficient to show that $LHS < RHS$ at $B = B'$. Plugging in B' and simplifying terms yields

$$\left(c_m - \alpha^2 K\sqrt{2c_m\left(2c_m - (1-g)\alpha^2 K\right)}\right)\left(-4c_m + \alpha^2 K + \sqrt{8c_m\left(2c_m - (1-g)\alpha^2 K\right)}\right) > 0. \tag{89}$$

Here, we get contradiction because the former term is positive whereas the latter term is negative. Therefore, for any $B' < B < \overline{B}$ firm value is decreasing at the point $q = q_I(B)$.

Proof of III.5: $\frac{dV(q_I(B) \leq q < q_L)}{dq}|_{q=q_L} = \frac{dV(q_L \leq q \leq 1)}{dq}|_{q=q_L}$

To proof that the claim is true, consider the firm value function in implicit form.

$$V(B' < B < \overline{B}, q_I(B) < q \leq q_L) = BV + FE$$

$$= \int_0^1 (\pi(q)2K - K)ds + \int_0^{s_I(q)} \left(\frac{(1-\pi(q))^2\alpha^2K^2}{2c_m} - gK \right) ds \tag{90}$$

$$V(B' < B < \overline{B}, q_L < q \leq 1) = BV + FE + IE$$

$$= \int_{s_L(q)}^1 (\pi(q)2K - K)ds + \int_{s_L(q)}^{s_I(q)} \left(\frac{(1-\pi(q))^2\alpha^2K^2}{2c_m} - gK \right) ds \tag{91}$$

Note, that the two functions only differ in the lower bound. Therefore, when calculating the derivative with respect to q all terms are the same, and thus cancel out, except for the change at the lower bound. The change at the lower bound evaluated at $s_L(q)$ calculates as

$$-(E[p \mid s_L(q)]2K - K) * \frac{\partial}{\partial q} s_L(q) - \frac{(1 - E[p \mid s_L(q)])^2\alpha^2K^2}{2c_m} - gK * \frac{\partial}{\partial q} s_L(q) \tag{92}$$

which we can rearrange to

$$\left(-(E[p \mid s_L(q)]2K - K) - \frac{(1 - E[p \mid s_L(q)])^2\alpha^2K^2}{2c_m} - gK \right) * \frac{\partial}{\partial q} s_L(q). \tag{93}$$

Now, we have already shown in the proof of Proposition (2) that the first term collapses to zero because of optimality of $s_L(q)$. Consequently, the claim must be true.

Proof of IV. For $\overline{B} < B$ the manager enters the public market whenever possible, i.e. $s_I(q, B) = 1/2$ or alternatively $q_I(B) = 0$. In the interval $0 \leq q < q_L$, firm value calculates as

$$
\begin{aligned}
&V(\overline{B} < B, 0 \leq q < q_L) \\
&= \int_0^{1/2} V(Pr \mid s, m^*, B) ds + \int_{1/2}^{1} V(Pu \mid s, B) ds \qquad (94) \\
&= \frac{K\left(\alpha^2 K\left(q^2 + 3q + 3\right) - 24 c_m g\right)}{48 c_m}
\end{aligned}
$$

which is strictly increasing in q. In the interval $q_L \leq q \leq 1$, firm value calculates as (94)$+IE$. Recall, that IE is also increasing in q and independent of B, so that firm value must be increasing in the interval $q_L \leq q \leq 1$ as well. In sum, for $\overline{B} \leq B$ firm value is strictly increasing in q. $\square$