DISCUSSION
PAPER SERIES
IN ECONOMICS
AND MANAGEMENT

An Elementary Approach to the Hold-Up Problem with Renegotiation

Urs Schweizer

Discussion Paper No. 00-01

GERMAN ECONOMIC ASSOCIATION OF BUSINESS
ADMINISTRATION - GEABA

An Elementary Approach to the Hold-Up Problem with Renegotiation -preliminary version -

Urs Schweizer
Department of Economics
University of Bonn
Adenauerallee 24
D-53113 Bonn
Germany
schweizer@uni-bonn.de

September 7, 2000

Abstract

This paper propagates a non-di¤erentiable and general approach to the hold-up problem with renegotiation. A simple condition is provided which necessarily must hold for an investment pro…le to be sustainable by a message contingent contract. A direction of investment is called non-harmful if the other party would never su¤er from an increase in investments by the …rst party. The e¢ cient investment pro…le fails to be sustainable as soon as, for one party at least, there exists a non-harmful direction of investment.

JEL classi…cation: D 82

1 Introduction

This paper deals with the mechanism design approach to the hold-up problem. There are two parties undertaking relationship-speci…c investments. After uncertainty has unraveled, some allocative decision has to be taken. Ex ante, the parties can sign any contract out of the general class of all message contingent contracts. Yet, if the chosen messages would lead to an ine¢cient decision, renegotiations are assumed to take place. By assumption, investments and the state of the world can be observed by both parties such that renegotiation takes place under complete information but they fail to be veri…able. The paper provides a simple condition which necessarily must hold for an investment pro…le to be sustainable by message contingent contracts with renegotiation.

Maskin and Moore [1999] were the …rst to formulate this problem in general. They give a characterization in terms of incentive compatibility constraints which, however, remain di¢cult to check. The present paper comes closer in spirit to Edlin and Reichelstein [1996] and, in particular, to Che and Hausch [1999] who have studied buyer-seller relationships with a one-dimensional quantity decision.

The present paper propagates an elementary and more general approach. It allows for multidimensional investments and decisions. No use of calculus is made. In particular, the set of decisions need not be connected. Therefore the results can be applied to hold-up problems as they arise in the property rights approach to the …rm (see, e.g. Grossman and Hart [1986] and Hart[1995]) In this approach, by ad-hoc assumption, ex-ante contracting is restricted to the choice of a non-contingent contract out of a …nite set of governance or ownership structures. The present paper also covers the setting of Roider [2000] who combines the choice of ownership structures with a continuous quantity choice.

The main result if applied to the e¢cient investment pro…le establishes that this pro…le fails to be sustainable if, for one party at least, there exists a direction of investment which is not harmful to the other party, i.e. the other party would receive a positive fraction of the surplus generated by additional investments of the …rst party. Moreover, for one-dimensional investments, it is shown that the second best contract is a non-contingent contract provided that, for both parties, investments are cooperative in the sense that increased spending by one party would lead to gains for the other party beyond its bargaining power. Finally, if only one of the parties invests then a potentially harmful decision must exist for the efficient decision to be sustainable. In this case, an option contract would provide the proper incentives.

The paper is organized as follows. The next section introduces the model. Section 3 contains the main results for the case of one-dimensional investments. Section 4 extends some of these findings to multi-dimensional investments. Section 5 concludes.

2 The model

Two parties $i = 1; 2$ choose investments $e_i \in E_i$. The sets E_i of feasible choices are assumed to be closed subsets of some finitely dimensional Euclidean spaces $\Re^{l_i}$. Investment profiles are denote by $e = (e_1; e_2) \in E = E_1 \times E_2$: Investment costs of party i are denoted by $c_i(e_i)$. After investment decisions have been taken, uncertainty $\omega \in \Omega$ unravels. For simplicity, the set Ω of states of the world is assumed to be finite ($\# \Omega = m$). The expectation operator with respect to the uncertain state of the world is denoted by $E_\omega[\cdot]$[1]. Any vector $\beta = (e; \omega) \in B = E \times \Omega$ is called a history of the hold-up problem. Histories can be observed by the two parties but fail to be verifiable in front of courts.

After the parties have learned the history, a decision $x \in X$ must be taken. The set of feasible decisions is assumed to be a subset of some Euclidean space, i.e. $X \subset \Re^n$.In some of the existing literature, this decision is assumed to concern a quantity choice, i.e. $X = [0; \infty)$ or $X = [0; 1]$: Yet, to also capture versions of the hold-up problem as studied in the property-rights approach to the theory of the firm (see introduction), our setting does not require the set X to be either one-dimensional or connected.

Profits (excluding investment costs and transfer payments) of party i amount to $p_i(\beta; x)$. The maximum social surplus

$$s(\beta) = \max_{x \in X} p_1(\beta; x) + p_2(\beta; x)$$

[1] The approach can easily be extended to infinite state spaces. All we require is that the expected value E_ω exists whenever use is made of the expectation operator.

is assumed to exist and to be finite for all histories. The first best investment profile which also is assumed to exist is denoted by

$$e^* \in \arg\max_{\beta \in B} E_\omega [s(\beta) - c_1(\beta) - c_2(\beta)].$$

Ex ante, i.e. before investment decisions are due, the parties sign a message contingent contract $\gamma = [M_1, M_2, x(m), t_1(m), t_2(m)]$ where $x(m) \in X$ and $t_1(m) + t_2(m) = 0$ denote the decision and the transfer payments at message profile $m \in M = M_1 \times M_2$. Let Γ denote the set of all message contingent contracts. This class is very general. It includes, in particular, all non-contingent contracts ($\# M_1 = \# M_2 = 1$) which play an important role in the property rights approach to the firm as well as all party i option contracts where only party i has a true choice as far as messages are concerned, i.e. $\# M_j = 1$ for party $j \neq i$ (see Segal and Whinston [1999] who study option contracts at length).

If, at history β and messages m, the contractual decision $x(m)$ fails to be efficient, renegotiations are assumed to take place and to lead to an efficient post-renegotiation solution. Post-renegotiation payoffs are denoted by

$$r_i(\beta, x(m)) + t_i(m) \tag{1}$$

where

$$r_i(\beta, x) = p_i(\beta, x) + \alpha_i(\omega) [s(\beta) - p_1(\beta, x) - p_2(\beta, x)]$$

is the post-renegotiation profit function of player i. The term in square brackets denotes the maximum social gain from renegotiation, out of which party i is assumed to get the share $\alpha_i(\omega)$ ($\alpha_i(\omega) \geq 0$, $\alpha_1(\omega) + \alpha_2(\omega) = 1$). While the bargaining power may depend on the state of the world, party i's share is exogenously fixed. Rearranging terms leads to

$$r_i(\beta, x) = (1 - \alpha_i(\omega)) p_i(\beta, x) + \alpha_i(\omega) [s(\beta) - p_j(\beta, x)]. \tag{2}$$

Notice that, for all histories β and all decisions x, it holds that

$$r_1(\beta, x) + r_2(\beta, x) = s(\beta). \tag{3}$$

Hence the message game with payoff functions (1) is a fixed-sum game such that, according to the Min-Max-Theorem, all its Nash equilibria are payoff equivalent. Let $m^\gamma(\beta) \in M$ denote one of these Nash equilibria (if there are several).

The payoff frontier $[R_1(\beta); R_2(\beta)]$ is called implementable if there exists a message contingent contract $\gamma \in \Gamma$ leading to a Nash equilibrium $m^\gamma(\beta)$ such that

$$R_i(\beta) = r_i(\beta; x(m^\gamma(\beta))) + t_i(m^\gamma(\beta))$$

holds for both parties $i = 1;2$. An incentive compatible mechanism asks parties separately to reveal the history, i.e. has message sets $M_1 = M_2 = B$ and it has telling the truth as a Nash equilibrium. It follows from the revelation principle (see Maskin and Moore [1999]) that the payoff frontier $[R_1(\beta); r_2(\beta)]$ is implementable if, and only if, there is an incentive compatible mechanism such that

$$R_i(\beta) = r_i(\beta; x(\beta;\beta)) + t_i(\beta;\beta)$$

holds for all histories $\beta \in B$. Finally, an investment profile $e^N \in E$ is called sustainable if there exists an implementable payoff frontier $[R_1(\beta); R_2(\beta)]$ such that

$$e_i^N \in \arg\max_{e_i \in E_i} E_\omega \left[R_i(e_i; e_j^N; \omega) \right] - c_i(e_i) \tag{4}$$

holds for $\{i;j\} = \{1;2\}$: Let E^N denote the set of sustainable investment profiles. Our main focus will be on this set.

3 One-dimensional investments

In this section, it is assumed that parties have one-dimensional investment choices only, i.e. $E_1; E_2 \subset \mathbb{R}$. Let $\Phi = \{\phi : \Omega \to X\} \subset \mathbb{R}^{mn}$ denote the set of all state contingent decisions ϕ and let

$$\pi_i(e;\phi) = E_\omega[r_i(e;\omega;\phi(\omega))] - c_i(e_i)$$

denote the net profit which party i would make if the investment profile were $e \in E$ and if the state contingent decision $\phi \in \Phi$ were implemented. For all $e_j \in E_j$; let

$$\varepsilon_i^+(e_j) = \sup_{e_i \in E_i; \phi \in \Phi} e_i$$

such that $\pi_i(e;\phi)$ is strictly monotonically decreasing for all $e_i' \geq e_i$ and

$$\varepsilon_i^-(e_j) = \inf_{e_i \in E_i; \phi \in \Phi} e_i$$

such that $\nu_i(e;\phi)$ is strictly monotonically increasing for all $e_i' \le e_i$. In other words, for $e_i \ge \varepsilon_i^+(e_j)$, $\nu_i(e;\phi)$ is strictly monotonically decreasing for all $\phi \in \Phi$. Similarly, for $e_i \le \varepsilon_i^-(e_j)$, $\nu_i(e;\phi)$ is strictly monotonically increasing for all $\phi \in \Phi$. Finally, let

$$E^{inf} = \{e \in E : \varepsilon_i^-(e_j) \le e_i \text{ for } i = 1,2\}$$

and

$$E^{sup} = \{e \in E : e_i \le \varepsilon_i^+(e_j) \text{ for } i = 1,2\}.$$

Then the following proposition can be established.

Proposition 1 For the set E^N of all sustainable investment profiles, it holds that $E^N \subset E^{inf} \cap E^{sup}$.

Proof. A choice function $f : B \to X$ is said to induce incentives e^N if

$$c_i(e_i^N) - c_i(e_i) \le E_\omega \left[r_i(e^N;\omega; f(e_i;e_j^N;\omega)) - r_i(e_i;e_j^N;\omega; f(e_i;e_j^N;\omega)) \right] \quad (5)$$

holds for all investments $e_i \in E_i$. We claim that, if $e^N \in E^N$ then there exists a choice function which induces e^N. In fact, it is well-known (see, e.g., Maskin and Moore [1999] or Segal and Whinston [1999]) that the payoff frontier $[R_1(\beta);R_2(\beta)]$ is implementable iff there exists a function $x : B \times B \to X$ such that

$$r_i(\beta';x(\beta;\beta')) - r_i(\beta;x(\beta;\beta')) \le R_i(\beta') - R_i(\beta) \le r_i(\beta';x(\beta';\beta)) - r_i(\beta;x(\beta';\beta))$$

holds for all histories β, $\beta' \in B$. To prove the claim, assume that $[R_1(\beta);R_2(\beta)]$ is implementable and provides incentives to invest e^N; i.e. (4) must hold for both parties. Then the claim is easily seen to hold for the choice function $f(e;\omega) = x(e^N;\omega;e;\omega)$.

To prove the proposition, assume first that $e_i^{sup} = \varepsilon_i^+(e_j^N) < e_i^N$. Let $e^{sup} = (e_i^{sup};e_j^N)$. It then follows from the definition of $\varepsilon_i^+(e_j^N)$ that $\nu_i(e^{sup};\phi) > \nu_i(e^N;\phi)$ must hold for all $\phi \in \Phi$; in particular for $\phi = f(e^{sup};\omega)$. But this contradicts (5). Therefore $\varepsilon_i^+(e_j^N) \ge e_i^N$ must hold for both parties.

Assume second that $\varepsilon_i^-(e_j^N) > e_i^N$. Then this leads to a contradiction in the same way as above. Therefore, $\varepsilon_i^-(e_j^N) \le e_i^N$ must hold for both parties. The proposition is established. ■

At first glance, the necessary condition of the proposition for an investment profile to be sustainable looks quite abstract. However, as we now want to show, the condition is closely related to the measures of cooperativeness as introduced by Che and Hausch [1999]. In order to establish this claim, for any state contingent decision $\phi \in \Phi$ and any investment profile $e \in E$, let us define

$$\begin{aligned} \pi_i(e;\phi) &= E_\omega\left[p_i(e;\omega;\phi(\omega))\right]; \\ \pi_j(e;\phi) &= E_\omega\left[p_j(e;\omega;\phi(\omega))\right]; \\ \rho_i(e;\phi) &= E_\omega\left[r_i(e;\omega;\phi(\omega))\right] \text{ and} \\ \sigma(e) &= E_\omega\left[s(e;\omega)\right]. \end{aligned}$$

The Greek letter simply expresses the expected value of the function with the corresponding Latin letter. To simplify, it is assumed that the bargaining power does not depend on the state of the world and that the expected net social surplus is a single-peaked function of both its arguments. More precisely, we assume the following:

Assumption SP

1. $\alpha_i(\omega) \equiv \alpha_i$, $0 < \alpha_i$, and $\alpha_1 + \alpha_2 = 1$
2. $\forall \mu \in [0;1]$; $\mu\sigma(e_i;e_j) - c_i(e_i)$ is strictly single-peaked [2] as a function of e_i, its peak being denoted by
$$B_i^{\mu}(e_j) = \arg\max_{e_i \in E_i} \mu\sigma(e_i;e_j) - c_i(e_i)$$
3. $B_i^{\mu}(e_j) < B_i^{1}(e_j)$ for all $\mu < 1$

Suppose party i would receive the fixed share $\mu < 1$ of the social surplus. Then (2) requires that its best response would come from maximizing a strictly single-peaked function whereas (3) requires that it would underinvest relative to the efficient response $B_i^1(e_j)$. This assumption would follow from the following more familiar assumption:

The sets E_i of feasible investments are intervals of the real line, $\sigma(e)$ and $c_i(e_i)$ are differentiable and strictly monotonically increasing functions, $\sigma(e)$ is a strictly concave and $c_i(e_i)$ a convex function of e_i, and the appropriate Inada conditions hold.

[2] The function is assumed to be strictly monotonically increasing to the left and strictly monotonically decreasing to the right of the peak.

While Che and Hausch [1999] have expressed their measures of cooperativeness in terms of derivatives, they can equally well be expressed in terms of differences. In fact, let

- $\alpha_i^c = \inf \alpha \in [0, 1]$ such that

  $$\alpha \left[\pi_j(e', \theta) - \pi_j(e, \theta)\right] \geq (1 - \alpha)\left[\pi_i(e', \theta) - \pi_i(e, \theta)\right]$$

  holds for all $e' = (e_i', e_j)$ and $e = (e_i, e_j) \in E$ where $e_i' > e_i$ and all $\theta \in \Theta$.

- Fix some small but positive fraction $0 < \varepsilon < \min[\alpha_1, \alpha_2]$. Let $\alpha_i^{nh} = \inf \alpha \in [0, 1]$ such that

  $$\begin{aligned} &\alpha \left[\pi_j(e', \theta) - \pi_j(e, \theta)\right] + (1 - \alpha - \varepsilon)\left[\sigma(e') - \sigma(e)\right] \\ \geq\ & (1 - \alpha)\left[\pi_i(e', \theta) - \pi_i(e, \theta)\right] \end{aligned}$$

  holds for all $e' = (e_i', e_j)$ and $e = (e_i, e_j) \in E$ where $e_i' > e_i$ and all $\theta \in \Theta$.

It follows that $\alpha_i^{nh} \leq \alpha_i^c$. Moreover, according to Che and Hausch, small values of α_i^{nh} and α_i^c mean that party i's investments are highly cooperative. The intuition behind these definitions is more easy to grasp if they are expressed in terms of the post-renegotiation profit functions. The following lemma whose proof immediately follows from (2), captures the essence.

Lemma 1 1. If $\alpha_i \geq \alpha_i^c$ then

$$\rho_i(e', \theta) - \rho_i(e, \theta) \leq \alpha_i\left[\sigma(e') - \sigma(e)\right] \tag{6}$$

2. whereas if $\alpha_i \geq \alpha_i^{nh}$ then

$$\rho_i(e', \theta) - \rho_i(e, \theta) \leq (1 - \varepsilon)\left[\sigma(e') - \sigma(e)\right] \tag{7}$$

holds for all $e' = (e_i', e_j)$ and $e = (e_i, e_j) \in E$ where $e_i' > e_i$ and all $\theta \in \Theta$.

It follows from the fixed-sum property (3) that (6) is equivalent to

$$\rho_j(e', \theta) - \rho_j(e, \theta) \geq \alpha_j\left[\sigma(e') - \sigma(e)\right];$$

i.e. if party i would increase its investments then the other party would benefit from that increase by a fraction not below its bargaining power. Therefore, if (6) holds, we say that party i's investments are cooperative (at the other party's investment level e_j). Similarly, it follows that (7) is equivalent to

$$\rho_j(e'; \phi) - \rho_j(e; \phi) \geq \alpha[\sigma(e') - \sigma(e)];$$

i.e. if party i would increase its investments then the other party would benefit from that increase by a small but positive share. Therefore, if (7) holds, we say that party i's investment are non-harmful (at the other party's investment level e_j).

Proposition 2 Under assumption SP, if, for one party i at least, investments are non-harmful at the other party's efficient level of investment e_j^* then the first best investment profile cannot be sustained, i.e. $e^* \notin E^N$.

Obviously, this proposition generalizes Proposition 3(i) of Che and Hausch. It easily follows from our Proposition 1. In order to establish this claim, use of the following lemma will be made.

Lemma 2 Suppose

1. $g(e_i) - f(e_i)$ is a monotonically increasing function of e_i;
2. $g(e_i) - c_i(e_i)$ is strictly single-peaked as a function of e_i,
3. $e_i^g = \arg\max g(e_i) - c_i(e_i)$ and
4. $e_i^g \leq e_i < e_i'$.

Then $f(e_i) - c_i(e_i) > f(e_i') - c_i(e_i')$.

First, we prove the lemma.

Proof. Due to strict single-peakedness, it holds that $g(e_i) - c_i(e_i) > g(e_i') - c_i(e_i')$. Moreover, due to monotonicity, it holds that $g(e_i) - f(e_i) \leq g(e_i') - f(e_i')$ from which the lemma follows immediately. ■

Second, we prove the proposition.

Proof. Let us apply the lemma to the functions $g(e_i) = (1 - \alpha)\sigma(e_i; e_j^N)$ and $f(e_i) = \rho_i(e_i; e_j^N; \phi)$. It follows from the above Lemma, (7) and assumption SP that $\rho_i(e_i; e_j^N; \phi)$ is strictly monotonically decreasing to the right of

$B_i^{1-\varepsilon}(e_j^*)$: Since this holds for all state contingent decisions ϕ, it follows that $\varepsilon_i^+(e_j^*) \leq B_i^{1-\varepsilon}(e_j^*) < B_i^1(e_j^*) = e_i^*$: Therefore $e^* \notin E^{sup}$ and hence, according to Proposition 1, cannot be sustained as was to be shown. ■

In order to generalize Proposition 3(ii) of Che and Hausch, let us introduce the following assumption which requires decisions to exist at which pre-renegotiation profits are vanishing. In Che and Hausch, this decisions correspond to the zero quantity which, at the same time, they identify with the Williamson contract, i.e. no ex-ante contract at all. In our assumption, the decisions can be different for the two parties. Moreover, they do not have to correspond to zero quantities.

Assumption 0

For both parties, there exists a decision $x_i^0 \in X$ such that pre-renegotiation profits are nil, i.e. $p_i(\beta; x_i^0) \equiv 0$: Hence, for post-renegotiation profits, it must hold that $r_i(\beta; x_i^0) = \alpha_i s(\beta)$:

Finally, let us define the set of investment profiles

$$E^{\alpha} = \{e \in E : e_i \leq B_i^{\alpha_i}(e_j) \text{ holds for } i = 1; 2\}$$

and let e^{**} be an investment profile such that

$$e_i^{**} = B_i^{\alpha_i}(e_j^{**}) \tag{8}$$

holds for both parties.

Proposition 3 Under assumptions SP and 0, if, for both parties, investments are cooperative at all investment levels of the other party then $E^N \subset E^{\alpha}$: Moreover, if reaction curves (8) are increasing and have a unique point of intersection e^{**} and if pre-renegotiation profit functions vanish at the same decision (i.e. $x_1^0 = x_2^0 = x^0$) then the investment profile e^{**} can be sustained by a non-contingent contract and must be the solution to the hold-up problem in the sense that it maximizes the expected net social surplus over all sustainable profiles.

Proof. Apply Lemma 2 to the functions $f(e_i) = \alpha_i\sigma(e_i; e_j^N)$ and $g(e_i) = \rho_i(e_i; e_j^N; \phi)$: It follows from the lemma, (6) and assumptions SP and 0 that $\rho_i(e_i; e_j^N; \phi)$ is strictly monotonically decreasing to the right of $B_i^{\alpha_i}(e_j)$: Since this holds for all state contingent decisions ϕ and since $\rho_i(e; x_i^0) = \alpha_i\sigma(e)$;

it follows that $B_i^{\alpha_i}(e_j) = \varepsilon_i^+(e_j)$ and, hence, that $E^\alpha = E^{sup}$. Therefore, it follows from Proposition 1 that $E^N \subset E^\alpha$ and the first part of the proposition is established.

As for the second part, let us assume that

$$e' \in \arg\max_{e \in E^\alpha} \sigma(e) - c_1(e_1) - c_2(e_2). \tag{9}$$

If, for one party i, it were the case that $e_i' < B_i^{\alpha_i}(e_j') < B_i^\infty(e_j')$ then, as follows from assumption SP, the social surplus at $e'' = (B_i^{\alpha_i}(e_j'); e_j')$ would strictly exceed the one at e'. Since $e'' \in E^N$ as follows from the assumption that reaction functions are increasing, this leads to a contradiction. Therefore, e' must be on both reaction curves. Since a unique point of intersection is assumed to exist, it must coincide with e^{**}. Finally, since $\rho_i(e; x') = \alpha_i \sigma(e)$, it follows that

$$e_i' = \arg\max_{e_i \in E_i} \rho_i(e; x') - c_i(e_i)$$

such that any non-contingent contract prescribing decision x' sustains the investment profile e^{**}. ■

To conclude this section, let us consider the case where only one of the parties, say $i = 1$, invests. We address the question under which conditions the first best investment profile can be sustained. A necessary condition, as follows from Proposition 1, would be that $\varepsilon_1^- \leq e_1^* \leq \varepsilon_1^+$. Remember that state contingent decisions enter the definitions of ε_1^- and ε_1^+. Obviously, it may cause difficulties to implement such state-contingent decisions such that this condition may well fail to be sufficient (unless there is no uncertainty, i.e. $\#\Omega = 1$). Therefore, let us extend the assumptions of Edlin and Reichelstein [1996] to the present setting.

Assumption ER:

There exists a decision $x^{ph} \in X$ such that

$$\rho_1(e_1'; x^{ph}) - \rho_1(e_1; x^{ph}) \geq \sigma(e_1') - \sigma(e_1) \tag{10}$$

holds for all $e_1' > e_1 \in E_1$ and such that $\nu_1(e_1; x^{ph})$ is a strictly single-peaked function of e_1.

Due to the fixed-sum property (3), the condition (10) is equivalent to $\rho_2(e_1'; x^{ph}) - \rho_2(e_1; x^{ph}) \leq 0$, i.e. party 2 would not benefit if party 1 were to increase its investments and if the decision x^{ph} were taken. In the spirit of our earlier terminology, such an investment could be called potentially harmful for the other party.

Edlin and Reichelstein derive from this condition and from assumption 0 that

$$e^0 = \arg\max_{e_1 \in E_1} \nu_1(e_1; x^0) \leq e_1^* \leq e^{ph} = \arg\max_{e_1 \in E_1} \nu_1(e_1; x^{ph})$$

must hold. They conclude from the intermediate value theorem that there must exist a decision $x \in X$ such that $\arg\max_{e_1 \in E_1} \nu_1(e_1; x) = e^*$. It then follows that any non-contingent contract prescribing decision x provides the efficient incentives to invest. Their approach requires further assumptions which we have not imposed. Without such assumptions, however, the first best investment level can still be sustained by the following party 2 option contract.

The ex-ante contract specifies decision x^{ph} but party 2 obtains the option to decision x^0 at strike price $S = \rho_1(e_1^*; x^0) - \rho_1(e_1^*; x^{ph})$. The option must be exercised before ω unravels.

Proposition 4 Under assumptions 0, SP and ER, the above party 2 option contract leads to the first best level of investments e^*.

Proof. Party 2 exercises the option iff

$$\rho_2(e_1; x^0) - S \geq \rho_2(e_1; x^{ph})$$

which, as follows from the fixed-sum property (3), is equivalent to

$$\rho_1(e_1; x^{ph}) - \rho_1(e_1; x^0) \geq S. \tag{11}$$

Notice that, by assumption, the option is exercised if party 2 is indifferent between exercising and abandoning it. If (11) holds and party 2 exercises the option then party 1 receives $\rho_1(e_1; x^0) - c_1(e_1) + S$ whereas if (11) is violated then party 1 receives $\rho_1(e_1; x^{ph}) - c_1(e_1)$. Therefore, party 1's net payoff can equivalently be summarized by either

$$\rho_1(e_1; x^0) - c_1(e_1) + \min\left[S; \rho_1(e_1; x^{ph}) - \rho_1(e_1; x^0)\right] \tag{12}$$

or

$$\rho_1(e_1; x^{ph}) - c_1(e_1) + \min\left[S + \rho_1(e_1; x^0) - \rho_1(e_1; x^{ph}); 0\right]. \tag{13}$$

If $e_1 \geq e_1^* \geq e^0$ then; as follows from assumptions SP, ER and (12), party 1's net payoff does not exceed $\rho_1(e_1; x^0) - c_1(e_1) + S$ such that, in this range, the optimum choice must be e_1^* leading to net payoff $\rho_1(e_1^*; x^0) - c_1(e_1^*) + S = \rho_1(e_1^*; x^{ph}) - c_1(e_1^*)$. If $e_1 \leq e_1^* \leq e^{ph}$ then; as follows from assumption ER and (13), party 1's net payoff does not exceed $\rho_1(e_1; x^{ph}) - c_1(e_1)$ such that, in this range, the optimum payoff cannot be higher than under the optimum in the other range. This establishes the proposition. ■

4 Multi-dimensional investments

Some of the results can easily be extended to the case of multi-dimensional investments as we now want to show. It is assumed that both parties have multi-dimensional investment choices $e_i \in E_i \subset \mathbb{R}^{l_i}$. Let $\Phi = \{\phi : \Omega \to X\} \subset \mathbb{R}^{mn}$ again denote the set of all state contingent decisions ϕ and let $\pi_i(e; \phi) = E_\omega[r_i(e; \omega; \phi(\omega))] - c_i(e_i)$ denote, as before, the net profit which party i would make if the investment profile were $e \in E$ and if the state contingent decision $\phi \in \Phi$ were implemented. Consider the set $\Delta_i = \{(d_1; d_2) \in \mathbb{R}^{l_1} \times \mathbb{R}^{l_2} : d_j = 0\}$ of investment directions for player i and let $\Lambda_i(e_j; d_i) = \{\lambda \in \mathbb{R} : e + \lambda d_i \in E\}$ denote the set of feasible investments at investment profile $e \in E$ in direction of $d_i \in \Delta_i$. Since E is assumed to be closed it follows that $\Lambda_i(e_j; d_i)$ must be closed as well. Moreover, since $e \in E$, it follows that $\Lambda_i(e_j; d_i)$ contains 0 and, hence, must be non-empty. For all $e_j \in E_j$, $d_i \in \Delta_i$ and $\phi \in \Phi$, let

$$\lambda_i(e_j; d_i; \phi) = \sup_{\lambda \in \Lambda_i(e_j; d_i)} \lambda$$

such that $\pi_i(e + \lambda' d_i; \phi)$ is strictly monotonically decreasing for all $\lambda' \geq \lambda$. Moreover, let

$$\varepsilon_i(e_j; d_i) = \sup_{\phi \in \Phi} \lambda_i(e_j; d_i; \phi):$$

Using this notation, the following result, which extends Proposition 1 to multi-dimensional investments, can be established.

Proposition 5 If the investment profile e^N is sustainable, i.e. if $e^N \in E^N$ then

$$\inf_{d_i \in \Delta_i} \sup_{\phi \in \Phi} \lambda_i(e_j^N; d_i; \phi) = \inf_{d_i \in \Delta_i} \varepsilon_i(e_j; d_i) \geq 0$$

must hold for both parties.

Proof. Since e^N is sustainable a choice function $f : B \to X$ must exist which induces incentives e^N, i.e. (5) holds for all investment choices $e_i \in E_i$.

To establish the proposition, assume the contrary which means that an investment direction $d_i \in \Delta_i$ must exist such that $\lambda_i^{sup} = \sup_{\phi \in \Phi} \lambda_i(e_j^N; d_i; \phi) < 0$. Let $e^{sup} = e^N + \lambda_i^{sup} d_i$. It follows from the definition of λ_i that $\nu_i(e^{sup}; \phi) > \nu_i(e^N; \phi)$ must hold for all $\phi \in \Phi$, in particular for $\phi = f(e^{sup}; \omega)$. But this contradicts (5). Therefore $\lambda_i^{sup} \geq 0$ must hold as was to be shown. ■

The measures of cooperativeness as introduced in the previous section can be extended to the multi-dimensional investments case as well. For some party i, fix a direction $d_i \in \Delta_i$, $d_i \neq 0$. For all $\lambda \in \Lambda_i = \Lambda_i(e_j; d_i)$ consider histories of the form $\beta = (e + \lambda_i d_i; \omega)$. Define

$$
\begin{aligned}
\pi_i(\lambda; \phi) &= E_\omega [p_i(\beta; \phi(\omega))]; \\
\pi_j(\lambda; \phi) &= E_\omega [p_j(\beta; \phi(\omega))]; \\
\rho_i(\lambda; \phi) &= E_\omega [r_i(\beta; \phi(\omega))]; \\
\gamma_i(\lambda) &= c_i(e + \lambda_i d_i) \text{ and} \\
\sigma(\lambda) &= E_\omega [s(\beta)].
\end{aligned}
$$

The Greek letter expresses the expected value of the function with the corresponding Latin letter in the direction of some given d_i.

If investments are one-dimensional then there exist only two directions of investments. In the multi-dimensional case, the situation becomes richer. In particular, if we consider the maximizers B_i^μ of the functions $\mu\sigma(\lambda) - \gamma_i(\lambda)$ for all $\mu \in [0; 1]$, the following cases may arise: first, B_i^μ may increase with μ, second, it may decrease with μ or, third, it may stay constant. The third case does not occur in the one-dimensional setting. In the following, this case need neither be taken into account. Moreover, if the investment direction d_i leads to the second case then $-d_i$ would lead to the first case. Therefore, in the following, we focus on the first case. Corresponding directions are called positive directions of investment. With this terminology at hand, assumption SP can be extended in the following way:

Assumption SP+

1. $\alpha_i(\omega) \equiv \alpha_i$, $0 < \alpha_i$, and $\alpha_1 + \alpha_2 = 1$

2. For all $\mu \in [0;1]$, $\mu\sigma(\theta) - \gamma_i(\theta)$ is strictly single-peaked as a function on θ, its peak being denoted by

$$B_i^{\mu} = \arg\max_{\theta \in \Theta_i(e_j;d_i)} \mu\sigma(\theta) - \gamma_i(\theta)$$

3. B_i^{μ} is a strictly increasing function for all $\mu \in [0;1]$:

This assumptions extends assumption SP in an obvious way. The measures of cooperativeness will directly be expressed in terms of the post-renegotiation profit functions (c.f. Lemma 1). We call the investment direction d_i cooperative and non-harmful if

$$\rho_i(\theta';\phi) - \rho_i(\theta;\phi) \leq \alpha_i\,[\sigma(\theta') - \sigma(\theta)]$$

and

$$\rho_i(\theta';\phi) - \rho_i(\theta;\phi) \leq (1-\varepsilon)\,[\sigma(\theta') - \sigma(\theta)]\,;$$

respectively (c.f. (6) and (7)), hold for all $\theta' > \theta \in \Theta_i(e_j;d_i)$. As before, ε is a given arbitrarily small but positive number. Proposition 2 can now be extended to the multi-dimensional case as follows:

Proposition 6 Under assumption SP, if for some party i, a positive investment direction exists which is not harmful to the other party then the first best profile cannot be sustained, i.e. $e^* \notin E^N$.

The proof makes use of Proposition 5 in exactly the same way as Proposition 2 does of Proposition 1. Therefore, the argument need not be repeated here. In principle, Proposition 3 could also be extended to the present case. Due to the great variety of investment directions, however, the analysis becomes more intricate and will not be pursued.

5 Concluding remarks

If an investment profile can be sustained by a message contingent contract then a choice function must exist which induces the post-renegotiation payoff frontier in the sense of equation (5). The existence of such a choice function has led to a simple condition which nessecarily must hold for an investment profile to be sustainable. While the condition can be formulated in very general terms, it will typically fail to be sufficient.

Segal and Whinston [1999] develop conditions which are sufficient for a payoff frontier to be implementable with renegotiation. Their approach parallels the one which Mirrlees [1971] had pioneered in a setting of one-dimensional private information. If applied to the hold-up problem, one-dimensional histories in the strict sense arise only in the case where only one of the parties invests and where there is no uncertainty. Yet, Segal and Whinston manage to provide some extensions to more interesting cases. To this end, they have to aggregate all investments into one dimension, leaving a second dimension for uncertainty. They provide a set of conditions such that any sustainable investment profile can also be sustained by a non-contingent contract and another set such that all sustainable investment profiles can also be sustained by an option contract. No attempt, however, is made to characterize the set of all sustainable investment profiles.

In their approach to implementation in general, choice functions which generate payoff frontiers play a crucial role. They must be distinguished from choice functions which induce incentives to invest in the sense of (5) and which are more closely tailored to the hold-up problem. Nevertheless, there is a relationship between the two concepts. In particular, if the set X of decisions is connected as Segal and Whinston assume then any sustainable payoff frontier $[R_1(\beta); R_2(\beta)]$ can be established by a choice function $f : B \to X$ in the sense that, for a given base β^0, $R_i(\beta) = R_i(\beta^0) + r_i(\beta; f(\beta)) - r_i(\beta^0; f(\beta))$ must hold for all histories β: Moreover, it is quite easy to characterize the choice functions which induce a given profile of investments. The only remaining problem concerns conditions which are sufficient for a payoff frontier, established by a choice function in the above sense, to be sustainable by some message contingent contract. While the incentive constraints would easily lead to such a condition, namely, for any two histories β and β', two decisions x and x' must exist such that

$$
\begin{aligned}
r_1(\beta'; x) - r_1(\beta; x) &\leq \left[r_1(\beta'; f(\beta')) - r_1(\beta'; f(\beta')) \right] - \\
\left[r_i(\beta; f(\beta)) - r_i(\beta^0; f(\beta)) \right] &\leq r_1(\beta'; x') - r_1(\beta; x');
\end{aligned}
$$

this condition is cumbersome and difficult to handle. It would be desirable to simplify this condition. Whether the Mirrlees or some other approach could be of help has to remain the subject of future research.

6 References

Che, Y.-C. and D. Hausch [1999], "Cooperative investments and the value of contracting," American Economic Review (89), 125-147.

Edlin, A. and S. Reichelstein [1996], "Holdups, standard breach remedies, and optimal investments," American Economic Review (86), 478-501.

Grossman, S.J. and O. Hart [1986], "The costs and bene…ts of ownership: a theory of vertical and lateral integration," Journal of Political Economy (94), 691-719.

Hart, O. [1995], Firms, Contracts, and Financial Structure. Clarendon Press, Oxford, U.K.

Maskin, E. and J. Moore [1999], "Implementation and renegotiation," Review of Economic Studies (66), 39-56.

Mirrlees, J. [1971], "An exploration in the theory of optimal income taxation," Review of Economic Studies (38), 175-208.

Roider, A. [2000], "On the foundations of the property rights theory of the …rm: cooperative investments and partial-enforcement contracts," Discussion Paper, University of Bonn.

Segal, I. and M.D. Whinston [1999], "The Mirrlees approach to mechanism design with renegotiation (with applications to hold-up and risk-sharing), Discussion Paper, Stanford University and Northwestern University.